University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2008/2009

MAPH 40240

Physical Meteorology

Extern examiner: Prof. Keith Shine

Head of School: Prof. Sean Dineen

Examiner: Dr. Rodrigo Caballero*

Time Allowed: 2 hours

Instructions for Candidates

Answer **any two (2)** of the following 3 questions. Each question carries 50 marks.
A list of values of physical constants can be found on the last page.

Instructions for Invigilators

Non-programmable calculators may be used during this examination. Tephigram charts will be handed to each candidate as part of the examination material.

Question 1

Consider the following two idealised midlatitude soundings:

	Sounding 1	
Pressure (hPa)	Temperature (°C)	Specific humidity (g/kg)
1000	6	1.5
750	-16	1.5
650	-14	2
240	-60	0.03

	Sounding 2	
Pressure (hPa)	Temperature (°C)	Specific humidity (g/kg)
1000	20	7
850	6	7
780	7	1.5
240	-60	0.02

(a) (*10 marks*) Plot temperature and dew point for each of the two soundings above, using separate tephigram charts. For each of the soundings, discuss the probable synoptic conditions that gave rise to the observed profiles.

(b) (*10 marks*) Give brief definitions of wet-bulb temperature T_w, equivalent temperature T_e and relative humidity r. For Sounding 1, use the tephigram to estimate T_w, T_e and r at 1000 hPa and at 750 hPa.

(c) (*10 marks*) In Sounding 1, identify a pressure layer that is likely occupied by cloud. Suppose this cloud layer is precipitating. Will the precipitation most likely be liquid (rain) or frozen (snow) when it reaches the ground? Explain your reasoning.

(d) (*5 marks*) Suppose Sounding 2 was taken in the early evening, and suppose that the sky is perfectly cloudless. Why is fog likely to form under these conditions, and at what temperature will it form?

(e) (*15 marks*) Is Sounding 2 absolutely stable, or conditionally unstable? Explain your answer. Suppose that Sounding 2 was taken in the morning, and strong insolation during the day increases surface temperature but does not change surface humidity. Estimate the surface temperature at which the sounding will become absolutely unstable. What kind of cloud will form once the profile achieves instability? At roughly what pressure level will cloud top be? By obtaining a very rough estimate of the CAPE, derive an estimate for the vertical speed of air parcels reaching cloud top".

Question 2

(a) (*10 marks*) The principle of energy conservation for an ideal gas undergoing a reversible adiabatic process can be written

$$c_v \frac{dT}{dt} + p\frac{d\alpha}{dt} = 0.$$

Show that this equation implies the conservation of potential temperature $\Theta = T(p/p_0)^{-R/c_p}$ in adiabatic transformations.

(b) (*10 marks*) State the Principle of Equipartition in classical mechanics, applied to an ideal gas. Using the principle, derive a relationship between the specific heat capacity at constant volume of a ideal gas and the number of degrees of freedom of each molecule in the gas.

(c) (*10 marks*) Show that the classical principle of equipartition, applied to an ideal diatomic gas, implies $R/c_p = 1/4$. At Earth-like temperatures, the actual value of R/c_p measured for a diatomic gas is closer to $2/7$. Explain how quantum mechanics accounts for this discrepancy.

(d) (*10 marks*) The Brunt-Väisälä frequency is given by

$$N = \sqrt{\frac{g}{\Theta}\frac{d\Theta}{dz}}.$$

Compute N for a layer of dry air having a constant temperature of 300 K.

(e) (*10 marks*) Repeat the calculation in (d) but assuming that the layer contains only helium, which is a *monatomic* gas with molecular mass $m = 6.64 \times 10^{-27}$ kg.

Question 3

(a) (*15 marks*) Figure 1 overleaf shows a Köhler diagram. Explain what this diagram shows and give a brief, qualitative account of the physical reasons for which the curve in the diagram takes the form shown.

(b) (*5 marks*) The curve in Fig. 1 has a maximum at around $r = 1.3\ \mu$m. How are drops with $r < 1.3\ \mu$m different from those with $r > 1.3\ \mu$m? If a subsaturated parcel of air, containing a large number of droplets all described by Fig. 1, is lifted beyond its lifting condensation level, what is the maximum supersaturation the parcel can achieve?

(c) (*10 marks*) Suppose you are watching the sunset. Conditions are such that, before reaching your eyes, the Sun's rays pass through a substantial layer of air containing haze droplets described by Fig. 1. Suppose the air in the layer is everywhere saturated (relative humidity is 100%). What colour would you expect to see? To answer this question, refer to Fig. 2 overleaf, showing the scattering efficiency Q_s of the droplets as a function of the scattering parameter $2\pi r/\lambda$. Neglect all effects other than scattering by the droplets

(d) (*10 marks*) The Beer-Lambert law for radiance passing through a purely scattering medium is

$$dI = -I\rho k_s ds.$$

Identify each of the terms in this equation. Give the definitions of *optical path* and *transmissivity*, and derive the relation between the two using the Beer-Lambert law as given above.

(e) (*10 marks*) Compute the transmissivity of the layer of air described in part (c) above, with respect to a monochromatic beam of wavelength 1 μm. Assume the layer is 10 km thick and contains a homogeneous droplet number density of 1 droplet/cm^3. Neglect all effects other than scattering by the droplets. Recall that

$$Q_s = \frac{m}{\pi r^2} k_s,$$

where m is the mass of each droplet.

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Figure 1:

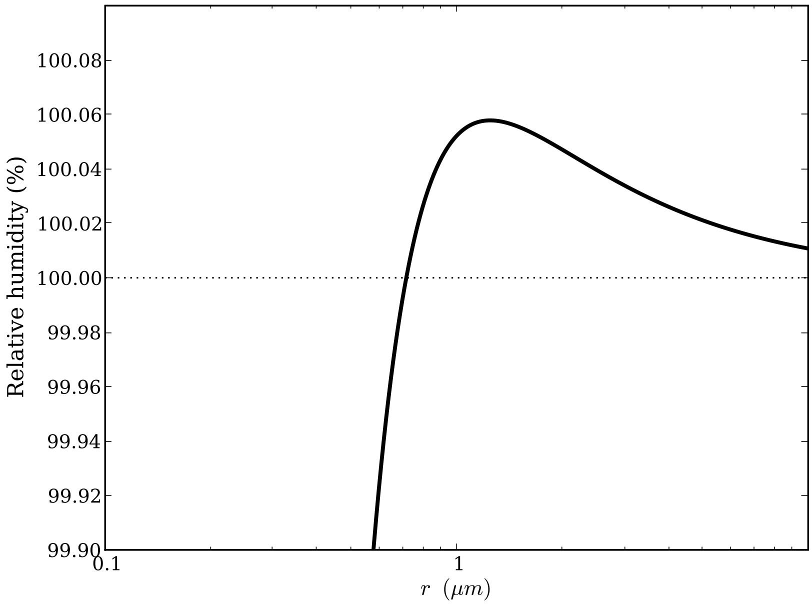
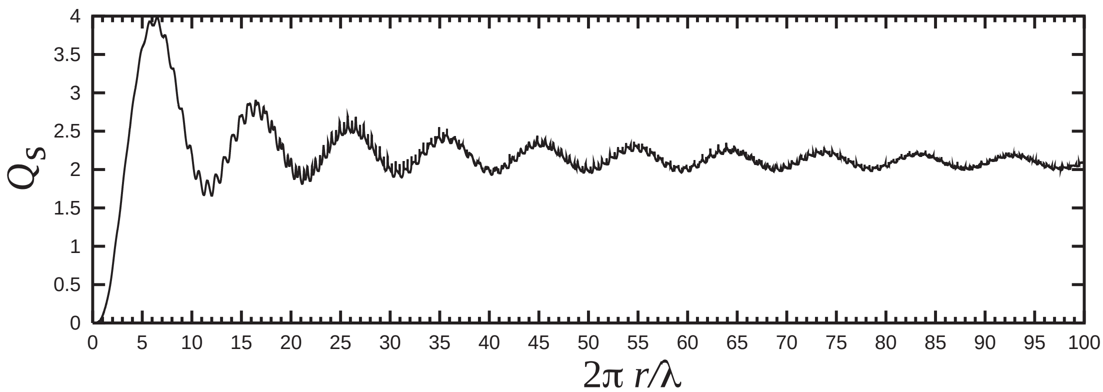


Figure 2:



Values of physical constants

Gravitational acceleration $g = 9.8$ m s^{-2}
Gas constant for dry air $R_d = 287$ J K^{-1} kg^{-1}
Specific heat capacity of dry air at constant pressure $c_{pd} = 1004$ J K^{-1} kg^{-1}
Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}
Boltzmann's constant $k = 1.38\times10^{-23}$ m^2 kg s^{-1} K^{-1}