University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2007/2008

MAPH 40240

Physical Meteorology

Extern examiner: Prof. Keith Shine

Head of School: Prof. Sean Dineen

Examiner: Dr. Rodrigo Caballero*

Time Allowed: 3 hours

Instructions for Candidates

All candidates must answer **Question 1**, which carries 50 marks. Candidates should then answer **two (2)** of the remaining 3 questions, each carrying 25 marks. A list of values of physical constants can be found on the last page.

Instructions for Invigilators

Non-programmable calculators may be used during this examination. Tephigram charts will be handed to each candidate as part of the examination material.

Question 1 ***(All candidates must answer this question)***

Consider the following idealized atmospheric sounding: n

Pressure (hPa)	Temperature (°C)	Relative humidity (%)
1000	30	65
700	10	
500	−5	
370	−20	
265	−40	
200	−57	
140	−57	

(a) (*5 marks*) State the definition (in words) of dew point temperature T_d, wet bulb temperature T_w, equivalent temperature T_e, lifting condensation level (LCL), level of free convection (LFC) and level of neutral buoyancy (LNB). Plot the sounding on the tephigram chart provided, and use it to determine the values of T_d, T_w, T_e, LCL, LFC and LNB for a parcel at 1000 hPa.

(b) (*5 marks*) Derive the mathematical relationship between pressure and height in a hydrostatic atmosphere. Estimate the height of the LCL of a surface air parcel. Discuss any approximations you make.

(c) (*10 marks*) The convective available potential energy is given by

$$\text{CAPE} = R_d \int_{p_{LNB}}^{p_{LFC}} (T_{vp} - T_{vs})\, d\ln p,$$

State the physical meaning of this quantity. Graphically estimate its value for the above sounding using the tephigram chart. Give an estimate for the vertical velocity of parcels arriving at the LNB. Why is this the *maximum* possible velocity?

(d) (*15 marks*) The parcels arriving at the LNB will overshoot, rising to some height above the LNB. Using the value of CAPE found in part (c), estimate the maximum height above the LNB that can be reached. Hint: note that the atmosphere above the LNB is isothermal.

Question 1 continued overleaf

(e) (*5 marks*) Suppose that solar heating is warming near-surface air without changing its dew point. Determine the temperature to which air at 1000 hPa must be heated in order for the profile to become absolutely unstable. What is the name given to this temperature? Estimate the CAPE once this temperature has been reached.

(f) (*10 marks*) Imagine that surface air from the sounding is forced to pass over a mountain range. If pressure is 850 hPa at the summit and the motion is pseudo-adiabatic (all condensation is immediately rained out), determine the temperature and relative humidity of the air once it is compressed back to 1000 hPa on the other side of the mountain.

Question 2

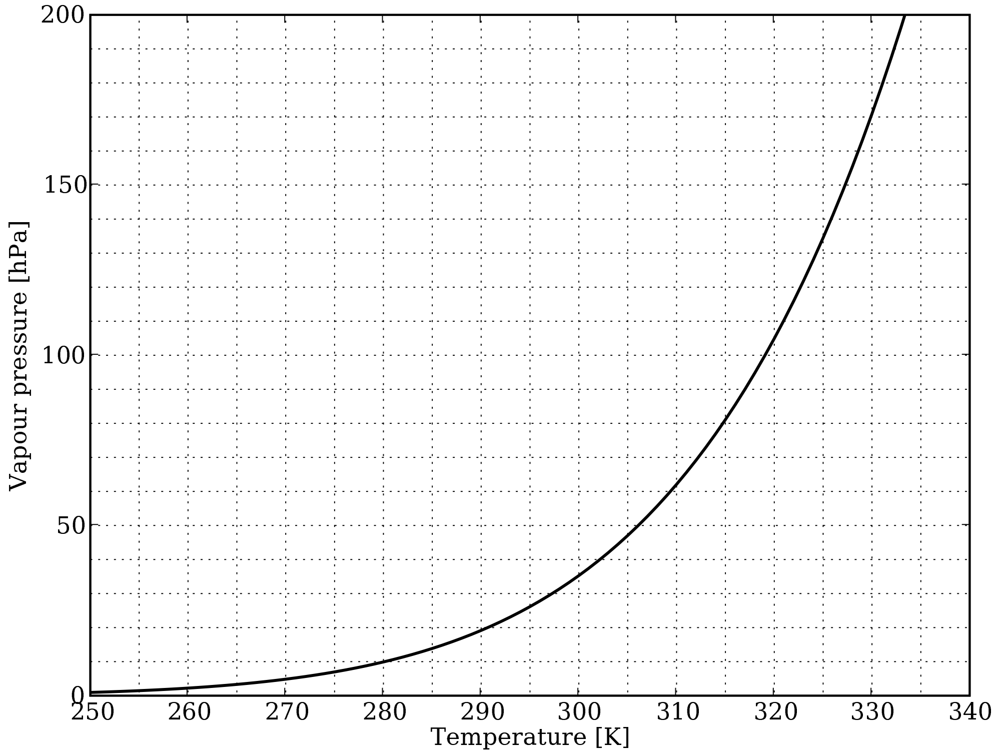

Figure 1:

Consider an airliner flying at a level where the temperature is 280 K and the relative humidity is 50%. Exhaust air exiting the jet engines has a temperature of 340 K and a water vapour pressure of 150 hPa (these are somewhat unrealistic values, used for the purpose of simplifying the exercise).

(a) (*5 marks*) Compute the buoyancy of air exiting the jet engines (you may neglect all moisture effects in this calculation).

(b) (*10 marks*) The air exiting the jet engines will rapidly cool by mixing with ambient air. Using the saturation vapour pressure curve in Figure 1, explain why this mixing leads to the formation of a contrail of *finite* length.

(c) (*10 marks*) If air in the contrail is cooling at a rate of 1 K s^{-1} and the aircraft is moving at a speed of 1000 km per hour, estimate the length of the contrail.

Question 3

(a) (*5 marks*) The psychrometric equation,

$$e = e_s(T_w) - \frac{p c_{pd}}{\epsilon \ell_v}(T - T_w)$$

is a relation between vapour pressure e and wet bulb temperature T_w, in terms of saturation vapour pressure e_s and the constant $c_{pd}/(\epsilon \ell_v) = 6.5 \times 10^{-4}$ Pa K^{-1}. If a whirling psychrometer, used at sea level, shows $T = 17.5°$C and $T_w = 7.5°$C, compute the ambient relative humidity (use the saturation vapour pressure curve in Figure 1 on the previous page).

(b) (*10 marks*) Kelvin's equation states

$$e_s(T, r) = e_{s\infty}(T) \exp\left(\frac{\gamma}{r\,T}\right),$$

where $e_s(T, r)$ is the saturation vapour pressure above a droplet of radius r and temperature T, $e_{s\infty}$ is the saturation vapour pressure above a flat surface, and $\gamma = 3.2 \times 10^{-7}$ m K is a constant. Explain in simple physical terms why the vapour pressure is higher above a droplet with smaller radius. If a droplet of radius $r = 10^{-8}$ m and temperature 17.5°C is placed in 5% supersaturated air at the same temperature, will the droplet grow or decay?

(c) (*10 marks*) State Raoult's law for the vapour pressure above a flat surface of water containing a certain number fraction of impurities. Explain why the vapour pressure is lower above impure water. Suppose that the water used in the psychrometer of part (a) is only 90% pure (i.e. only 9 out of 10 molecules is water). If you erroneously believed the water to be pure, would you overestimate or underestimate the ambient vapour pressure? By how much?

Question 4

(a) (*5 marks*) Define the *absorptivity* and *emissivity* of a body. State Kirchhoff's law linking these two quantities.

(b) (*10 marks*) State Stefan's law for the total flux of radiant energy emitted by a black body at temperature T. Assuming the atmosphere to be hydrostatic with a surface pressure of 1000 hPa, and taking the atmosphere's specific heat capacity as 1004 J K^{-1} kg^{-1} and the effective emission temperature as 255 K, estimate the mean cooling rate of the atmosphere due to longwave emission to space. Express your result in units of K day^{-1}.

(c) (*10 marks*) Derive the relation between absorptivity and optical path τ. Compute the radiative flux emitted by a slab of air with optical path $\tau = 1$ and temperature 255 K. Throughout this question, you may assume the optical path is independent of wavelength and neglect any scattering effects.

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Values of physical constants

Gravitational acceleration $g = 9.8$ m s^{-2}
Gas constant for dry air $R_d = 287$ J K^{-1} kg^{-1}
Specific heat capacity of dry air at constant pressure $c_{pd} = 1004$ J K^{-1} kg^{-1}
Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}