University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER II EXAMINATION 2010/2011

ACM 40520

Numerical Weather Prediction

Extern examiner: Prof Peter Clark

Head of School: Prof Mícheál Ó Searcóid

Examiner: Prof Peter Lynch*

Time Allowed: 2 hours

Instructions for Candidates

Answer **three (3)** questions.
All questions carry equal marks.

Please avoid the use of red ink on the answer books.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1

(a) (*8 marks*) Given two independent, unbiassed temperature observations T_1 and T_2 with (unknown) errors ε_1 and ε_2, and (known) variances σ_1^2 and σ_2^2, derive an expression for the best linear unbiassed estimate (BLUE) of the temperature. State all assumptions that you make.

(b) (*10 marks*) Assuming $T_1 = 14°\text{C}$, $T_2 = 16°\text{C}$, $\sigma_1 = 1°\text{C}$ and $\sigma_2 = 3°\text{C}$, find the temperature that minimizes the least squares error, and also find its variance.

(c) (*2 marks*) Show that the temperature that minimizes the *cost function*

$$J(T) = \frac{1}{2}\left[\left(\frac{T - T_1}{\sigma_1}\right)^2 + \left(\frac{T - T_2}{\sigma_2}\right)^2\right]$$

yields the same estimate of T as the least squares estimate. Briefly discuss the consequences of this equivalence in the context of optimal interpolation (OI) analysis and variational assimilation.

Question 2

Consider the ordinary differential equation for a function of time, $u = u(t)$,

$$\frac{\mathrm{d}u}{\mathrm{d}t} = F(u) \tag{1}$$

where $F(u)$ is an arbitrary nonlinear function of u.

(a) (*8 marks*)

Write down the finite difference approximations to (1) using each of the following schemes:

(*i*) Euler or forward scheme

(*ii*) Backward or fully implicit scheme

(*iii*) Centered implicit scheme

(*iv*) Leapfrog scheme

(b) (*10 marks*)

The Adams-Bashforth scheme for the equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = i\omega u \tag{2}$$

(the linear oscillation equation) may be written

$$\frac{u^{n+1} - u^n}{\Delta t} = i\omega \left(\tfrac{3}{2}u^n - \tfrac{1}{2}u^{n-1}\right) . \tag{3}$$

Perform a stability analysis and show that the physical solution and the computational solution have amplitudes

$$|\rho_+| = 1 + \tfrac{1}{4}(\omega\Delta t)^4 \qquad \text{and} \qquad |\rho_-| = \tfrac{1}{2}(\omega\Delta t)$$

respectively.

(c) (*2 marks*) Comment on the practical utility of the Adams-Bashforth scheme for NWP and for climate modelling.

Question 3

(a) (*6 marks*)

Consider a test function of time, $f(t)$, of the form

$$f(t) = \alpha_1 \cos(\omega_1 t - \psi_1) + \alpha_2 \cos(\omega_2 t - \psi_2)$$

where $|\omega_1| \ll |\omega_2|$ and α_1, α_2, ψ_1 and ψ_2 are arbitrary (constant) amplitudes and phases. Describe how, by taking the Laplace transform of $f(t)$ and applying a modification of the inversion operator, the high-frequency component $\alpha_2 \cos(\omega_2 t - \psi_2)$ may be filtered out of the solution.

(b) (*10 marks*)

The atmospheric equations of motion may be written in symbolic form

$$\dot{\mathbf{X}} + \mathbf{L}\mathbf{X} + \mathbf{N}(\mathbf{X}) = \mathbf{0} \tag{4}$$

where $\mathbf{X}$ is the state vector, $\mathbf{L}$ is a linear operator (matrix) and $\mathbf{N}(\mathbf{X})$ is a nonlinear vector function.

Show how this system may be written in normal mode form (assume $\mathbf{LE} = \mathbf{E}\boldsymbol{\Lambda}$ where $\boldsymbol{\Lambda}$ is a diagonal matrix), so that rotational and gravity wave components can be distinguished.

Apply the Laplace transform to derive an algebraic expression for the transform of the solution. Describe how, by a modification of the inversion operator, a solution comprising the low-frequency components of the solution of (4) may be obtained.

(c) (*4 marks*)

Describe the definition of the Laplace transform that is appropriate in order to combine this approach with a semi-Lagrangian advection scheme.

Comment on the computational efficiency of the scheme for (i) finite-difference models and (ii) spectral models.

Question 4

Consider the barotropic vorticity equation on a sphere

$$\frac{\partial \zeta}{\partial t} + \frac{2\Omega}{a^2}\frac{\partial \psi}{\partial \lambda} + \frac{1}{a^2}J(\psi, \zeta) = 0 \tag{5}$$

where the Jacobian term is defined as

$$J(\psi, \zeta) = \left[\frac{\partial \psi}{\partial \lambda}\frac{\partial \zeta}{\partial \mu} - \frac{\partial \psi}{\partial \mu}\frac{\partial \zeta}{\partial \lambda}\right],$$

$\mu = \sin\phi$ and all other notation is standard.

(a) (*6 marks*) Assuming that the streamfunction may be expanded in spherical harmonics

$$\psi(\lambda, \mu, t) = \sum_{n=0}^{N}\sum_{m=-n}^{n} \psi_n^m(t) Y_n^m(\lambda, \mu),$$

show that the vorticity $\zeta(\lambda, \mu, t)$ has a similar expansion, with

$$\zeta_n^m = -\frac{n(n+1)}{a^2}\psi_n^m.$$

(b) (*8 marks*) Give a *general argument* leading to the conclusion that the evolution of the spectral coefficients is governed by a set of ode's of the form

$$\frac{d\zeta_n^m}{dt} = -i\sigma_n^m \zeta_n^m + \tfrac{1}{2} i \textstyle\sum_{k\ell rs} I_{nrs}^{mk\ell}\zeta_r^k \zeta_s^\ell$$

where $\sigma_n^m = -2\Omega m / n(n+1)$ is the Rossby-Haurwitz wave frequency (it is not necessary to derive explicit expressions for the coefficients $I_{nrs}^{mk\ell}$).

(c) (*6 marks*) Describe, in outline, the *transform method*, and give the reason why it is considered superior to the interaction coefficient method indicated in part (b) above.

Question 5

The one-dimensional advection equation may be written in either the Eulerian form or the Lagrangian form:

$$\frac{\partial Y}{\partial t} + u\,\frac{\partial Y}{\partial x} = 0 \qquad \text{or} \qquad \frac{\mathrm{d}Y}{\mathrm{d}t} = 0\,.$$

(a) (*7 marks*) Describe the procedure used to develop a (semi-Lagrangian) finite difference approximation to the Lagrangian form of the equation.

(b) (*8 marks*) Assuming that linear spatial interpolation from surrounding points is used to evaluate the solution at the departure point, show that the semi-Lagrangian scheme is stable irrespective of the time step.

(c) (*5 marks*) Discuss the consequences of unconditional stability for operational numerical weather prediction. Does unconditional stability of the scheme enable an arbitrarily large time step to be used in practice, or are there other considerations limiting it?

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