University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2011/2012

ACM 40460

Dynamic Meteorology

External examiner: Professor Peter A Clark

Head of School: Dr Patrick Murphy

Lecturer: Professor Ray Bates*

Time Allowed: 2 hours

Instructions for Candidates

Answer **three** (3) questions.
Please use separate answer book for each question.
Please *do not* use a red pen.

All questions carry equal marks. Total: 75 marks.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1.

(a) Write down the equation of state and the hydrostatic equation for a dry atmosphere, explaining the meaning of the symbols.

(b) Given the thermodynamic equation for dry air in the form

$$c_p \frac{D \ln T}{Dt} - R \frac{D \ln p}{Dt} = \frac{J}{T} \qquad (1.1)$$

where the notation is conventional, show that the equation can be rewritten in terms of potential temperature (θ) as

$$\frac{D\theta}{Dt} = \frac{\theta}{c_p T} J \qquad (1.2)$$

(c) Using the non-hydrostatic vertical momentum equation (with Coriolis, curvature and frictional terms omitted) and the adiabatic form of (1.2), show that small-amplitude buoyancy oscillations about a resting basic state have frequency N, where

$$N \equiv \left[g \frac{d \ln \theta_0}{dz} \right]^{1/2} \qquad (1.3)$$

Calculate the period of the buoyancy oscillation if $N = 1.0 \times 10^{-2} s^{-1}$.
Note: The basic state, whose potential temperature is $\theta_0(z)$, is assumed to be hydrostatically balanced and stably stratified. In the analysis, use the parcel method, where the pressure of a displaced parcel adjusts instantaneously to the pressure of the environment.

Question 2.

(a) Using the hydrostatic equation and the equation of state for an ideal gas, show that the difference in geopotential between two isobaric surfaces with pressures p_0 and p_1 (where $p_0 > p_1$) is

$$\Phi_1 - \Phi_0 = R\langle T \rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (2.1)$$

where R is the gas constant and $\langle T \rangle$ is the mean temperature of the intervening layer.

(b) Using the hydrostatic equation and the definition of the geostrophic wind $(\vec{V}_g)$, show that the difference in $\vec{V}_g$ between levels p_1 and p_0 is

$$\vec{V}_g(1) - \vec{V}_g(0) = \frac{R}{f}\vec{k} \times \vec{\nabla}\langle T\rangle \ln\left(\frac{p_0}{p_1}\right) \qquad (2.2)$$

(c) Assuming $\langle T\rangle$ for the 1000-850 hPa layer in the neighbourhood of a point on the 45° latitude circle is given by

$$\langle T\rangle = 280K - \left(5\times 10^{-3}\frac{K}{km}\right)y \qquad (2.3)$$

where y is the distance northward from this point, use (2.1) and (2.2) to find ` the thickness of the layer and the geostrophic wind difference across the layer at the point in question. [R = 287 J/(kgK), g = 9.81m/s^2]

Question 3.

a) The potential vorticity equation for the shallow water model with a free surface and a flat bottom on an f-plane can be written in conventional notation as

$$\frac{D}{Dt}\left[\frac{\varsigma + f_0}{\Phi}\right] = 0 \qquad (3.1)$$

Show that the linearized version of (3.1) for small perturbations about a state of rest can be written

$$\frac{\partial}{\partial t}\left(\varsigma' - \frac{f_0}{\overline{\Phi}}\Phi'\right) = 0 \qquad (3.2)$$

b) At an initial time t=0 the surface height of the above shallow water model is given by

$$h = H + h_0,\ x < 0 \qquad (3.3)$$

$$h = H - h_0,\ x > 0 \qquad (3.4)$$

and the initial velocity is zero everywhere. Using (3.2) show that the equations determining the surface height perturbations at t=∞, when the system has adjusted to geostrophic balance, are

$$\frac{d^2}{dx^2}(h' - h_0) = \frac{1}{R^2}(h' - h_0),\ x < 0 \qquad (3.5)$$

$$\frac{d^2}{dx^2}(h' + h_0) = \frac{1}{R^2}(h' + h_0),\ x > 0 \qquad (3.6)$$

where $R = \sqrt{\overline{\Phi}} / f_0$.

(c) Solve (3.5) and (3.6) subject to the appropriate boundary conditions to obtain the surface height at t=∞.

Question 4.

(a) The governing momentum and continuity equations for the shallow water model with a rigid horizontal lid, mean depth H, bottom topography of height $h_s(x,y)$ and fluid depth h(x,y) on a β-plane are given by

$$\frac{Du}{Dt} = fv - \frac{\partial \Phi_T}{\partial x} \qquad (4.1)$$

$$\frac{Dv}{Dt} = -fu - \frac{\partial \Phi_T}{\partial y} \qquad (4.2)$$

$$\frac{Dh}{Dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad (4.3)$$

Derive the quasi-geostrophic form of these equations.
Note: use constant-f geostrophy in defining the geostrophic flow.

(b) Hence show that the quasi-geostrophic potential vorticity equation for this model can be written

$$\frac{D_g}{Dt}\left[\zeta_g + f + \frac{f_0}{H} h_s\right] = 0 \qquad (4.4)$$

Question 5.

(a) The quasi-geostrophic potential vorticity equation for frictionless adiabatic flow in Z [= - H ln(p/p_s)] coordinates on a β-plane (f= f_0 +βy) can be written

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \bullet \vec{\nabla}\right) q = 0 \qquad (5.1)$$

where

$$q = \nabla^2 \psi + f + \frac{1}{\rho_0}\frac{\partial}{\partial Z}\left(\frac{f_0^2}{N^2}\rho_0 \frac{\partial \psi}{\partial Z}\right) \qquad (5.2)$$

In the above, ψ is the geostrophic streamfunction, ∇^2 is the horizontal Laplacian, $\vec{V}_g = \vec{k} \times \vec{\nabla}\psi$, N is the buoyancy frequency and ρ_0 is the reference density, given by

$$\rho_0 = \rho_s \exp(-Z/H) \qquad (5.3)$$

with ρ_s and H both constants.

Show that the linearized form of (5.1) for a small perturbation about a constant mean wind $\bar{u}$ can be written

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right) q' + \beta \frac{\partial \psi'}{\partial x} = 0 \qquad (5.4)$$

where

$$q' = \nabla^2 \psi' + \frac{1}{\rho_0} \frac{\partial}{\partial Z} \left(\frac{f_0^{\,2}}{N^2} \rho_0 \frac{\partial \psi'}{\partial Z} \right) \qquad (5.5)$$

Hint: assume $\psi = -\bar{u} y + \psi'(x, y, Z, t)$.

(b) Consider the case of a perturbation forced by the mean wind blowing over topography of the form

$$h = h_s \cos ly \, e^{ikx} \qquad (5.6)$$

Assuming N is constant, show that a steady-state solution to (5.4) that is oscillatory solution in Z (and can therefore propagate wave energy vertically) is possible only if the mean wind $\bar{u}$ lies between zero and an upper limit U_c given by

$$U_c = \frac{\beta}{\left(k^2 + l^2\right) + \frac{1}{4L_R^{\,2}}} \qquad (5.7)$$

where L_R (=NH/f_0) is the Rossby radius of deformation.
Hint: Assume the solution has the form

$$\psi' = \Psi(Z) \exp(Z/2H) \cos ly \, \mathrm{e}^{\mathrm{ikx}} \qquad (5.8)$$

Note: you are not required to make explicit use of (5.6) to find the full solution satisfying the lower boundary condition here. The form of the topography is given only to indicate why the topographically forced solution should have the x and y-dependence given in (5.8).

(c) Find the value of U_c if the β-plane is tangent to the Earth at 45° latitude, L_R = 1000 km, $l = 0$ and the x-wavelength is that corresponding to zonal wavenumber 1 at the latitude of tangency. (Ω = 7.29 x 10^{-5} s^{-1}, a = 6370 km.)

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