University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER I EXAMINATION 2010/2011

ACM 40460

Dynamic Meteorology

Extern examiner: Professor Peter A Clark

Head of School: Professor Mícheál Ó Searcóid

Lecturer: Professor Ray Bates*

Time Allowed: 2 hours

Instructions for Candidates

Answer **three (3)** questions.
All questions carry equal marks. Total: 75 marks.
Please do not use red pen on the answer books.
Please use separate answer book for each question.

Instructions for Invigilators

Non-programmable calculators may be used during this examination.

Question 1. [Marks. (a): 5. (b): 10. (c): 10]

(a) Define potential temperature (θ). Show that when θ is constant with height (z) the lapse rate of temperature is given by

$$\Gamma_d \equiv -\frac{dT}{dz} = \frac{g}{c_p} \qquad (1.1)$$

where g is the acceleration of gravity and c_p is the specific heat of dry air at constant pressure. (Assume the atmosphere is hydrostatically balanced.)

(b) Given the thermodynamic energy equation in the form

$$c_v \frac{DT}{Dt} + p\frac{D\alpha}{Dt} = J \qquad (1.2)$$

where c_v is the specific heat of dry air at constant volume, α is the specific volume and J the diabatic heating rate per unit mass, show that this equation can be written in terms of potential temperature as

$$\frac{D\theta}{Dt} = \frac{\theta}{c_p T} J \qquad (1.3)$$

(Use the standard thermodynamic relationship $R = c_p - c_v$).

Hint: You may work backwards from (1.3) using the definition of θ if you wish.

(c) Show that in the isobaric coordinate system eq. (1.2) can be written in the form

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - S_p \omega = \frac{J}{c_p} \qquad (1.4)$$

where

$$S_p = \frac{\alpha}{c_p} - \frac{\partial T}{\partial p} \qquad (1.5)$$

Note: $\omega \equiv Dp/Dt$.

Question 2. [Marks. (a): 5. (b): 13. (c): 7]

a) State the assumptions used in constructing the shallow water model with a free surface of height $h_T(x,y,t)$, bottom topography $h_s(x,y)$ and fluid depth $h(x,y,t)$ on a β-plane.

b) Starting from the primitive equations of motion on a β-plane and using the above assumptions, show that the governing equations for the model in question are

$$\frac{Du}{Dt} = fv - \frac{\partial \Phi_T}{\partial x} \qquad (2.1)$$

$$\frac{Dv}{Dt} = -fu - \frac{\partial \Phi_T}{\partial y} \qquad (2.2)$$

$$\frac{Dh}{Dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad (2.3)$$

where $\Phi_T = gh_T$.

c) Derive the linearized form of the above equations for small perturbations about a state of rest for the case where $h_s = 0$ and $\beta = 0$. Hence show the existence of 1D gravity-inertia wave solutions with phase speed

$$c = \pm\left(gH + \frac{f_0^{\,2}}{k^2}\right)^{1/2} \qquad (2.4)$$

where H is the mean depth of the fluid and k is the wavenumber in x.

<u>Question 3</u>. [Marks. (a): 11. (b): 14]

a) The quasi-geostrophic potential vorticity equation for the free-surface shallow water model on a β-plane ($f = f_0 + \beta y$) is given by

$$\frac{D_g}{Dt}\left[\nabla^2\psi + f - \frac{\psi}{\lambda^2}\right] = -\frac{f_0}{\Phi_0}\frac{D_g\Phi'_s}{Dt} \qquad (3.1)$$

where D_g / Dt is the material derivative following the geostrophic motion, ψ is the geostrophic streamfunction, ∇^2 is the horizontal Laplacian, $\lambda = \sqrt{\Phi_0} / f_0$, Φ_0 is the mean geopotential of the free surface and Φ'_s is the geopotential of the orography (assumed small by comparison with Φ_0).

Using the linearized perturbation form of (3.1) for a resting basic state with $\Phi'_s=0$, derive the phase speed for a 1D Rossby wave propagating in the x-direction in this model.

b) Again using (3.1) but neglecting β, derive the solution for the perturbation streamfunction ψ' in the case of steady motion forced by a mean current $\bar{u}$ blowing over orography of the form

$$\Phi'_s = \mathrm{Re}\left[\hat{\Phi}_s \exp(ikx)\right] \qquad (3.2)$$

Hint: Since the forcing has spatial dependence of the form exp(ikx), the solution may also be assumed to have spatial dependence of this form.

<u>Question 4.</u> [Marks. (a): 6. (b): 6. (c):13]

(a) Show that in a hydrostatic atmosphere the internal and gravitational potential energies are proportional and that the sum of these two forms of energy (i.e., the total potential energy – TPE) can be written

$$TPE = \left(\frac{c_p}{c_v}\right)E_I \qquad (4.1)$$

where c_p and c_v are the specific heats of air at constant pressure and constant volume, respectively, and E_I is the internal energy.

(b) Show that the TPE of a unit column of atmosphere of uniform potential temperature θ extending from the surface ($p=p_0$) to the top of the atmosphere ($p=0$) is

$$TPE = \frac{c_p}{g}\frac{p_0\theta}{\kappa+1} \qquad (4.2)$$

where $\kappa = R/c_p$ and g is the acceleration of gravity.

(c) Consider two air masses of uniform potential temperatures θ_1 and θ_2 ($\theta_2 > \theta_1$) which are separated by a vertical partition. Each air mass occupies a horizontal area A and extends from the surface (where $p=p_0$ for each) to the top of the atmosphere. Show that the available potential energy for this system is given by

$$APE = \frac{c_p}{g}\frac{p_0}{\kappa+1}\left(1-\frac{1}{2^\kappa}\right)(\theta_2-\theta_1)A \qquad (4.3)$$

Question 5. [Marks. (a): 15. (b): 4. (c): 6]

(a) Given the following governing equations for the two-layer model of baroclinic instability

$$\left(\frac{\partial}{\partial t}+U_1\frac{\partial}{\partial x}\right)\frac{\partial^2\psi_1'}{\partial x^2}+\beta\frac{\partial\psi_1'}{\partial x}=\frac{f_0}{\delta p}\omega_2' \qquad (5.1)$$

$$\left(\frac{\partial}{\partial t}+U_3\frac{\partial}{\partial x}\right)\frac{\partial^2\psi_3'}{\partial x^2}+\beta\frac{\partial\psi_3'}{\partial x}=-\frac{f_0}{\delta p}\omega_2' \qquad (5.2)$$

$$\left(\frac{\partial}{\partial t}+U_m\frac{\partial}{\partial x}\right)\left(\psi_1'-\psi_3'\right)-U_T\frac{\partial}{\partial x}\left(\psi_1'+\psi_3'\right)=\frac{\sigma\delta p}{f_0}\omega_2' \qquad (5.3)$$

where $U_m = (U_1+U_3)/2$ and $U_T=(U_1-U_3)/2$, show that the rate of change of the sum of the kinetic and available potential energies is

$$\frac{d}{dt}(K'+P')=4\lambda^2U_T\overline{\psi_T\frac{\partial\psi_m}{\partial x}} \qquad (5.4)$$

where $\psi_m = \left(\psi_1' + \psi_3'\right)/2$, $\psi_T = \left(\psi_1' - \psi_3'\right)/2$, $\lambda^2 = (f_0)^2/\left[\sigma(\delta p)^2\right]$ and $\overline{(\;)}$ denotes an average over the wavelength of the disturbance.

(b) Discuss the energetics of baroclinic waves qualitatively using a box diagram involving $\overline{P}$ (the mean available potential energy), P' and K'.

(c) Discuss the physical mechanism of baroclinic instability in terms of the slope of particle trajectories ('the wedge of instability').

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