Royal Irish Academy

The Navier-Stokes Equations:

The Key to Modern Weather Forecasting.

Peter Lynch
Met Éireann
Dublin
19 June, 2003
Peter.Lynch@met.ie

Outline of Talk

- ***Stokes' Contributions (c. 1850)***
- ***The Pre-history of Numerical Weather Prediction (c. 1900)***
- ***The ENIAC Integrations (c. 1950)***
- ***Modern Computer Forecasting (c. 2000)***

C. L. M. H. Navier, 1785–1836

Claude Louis Marie Henri Navier

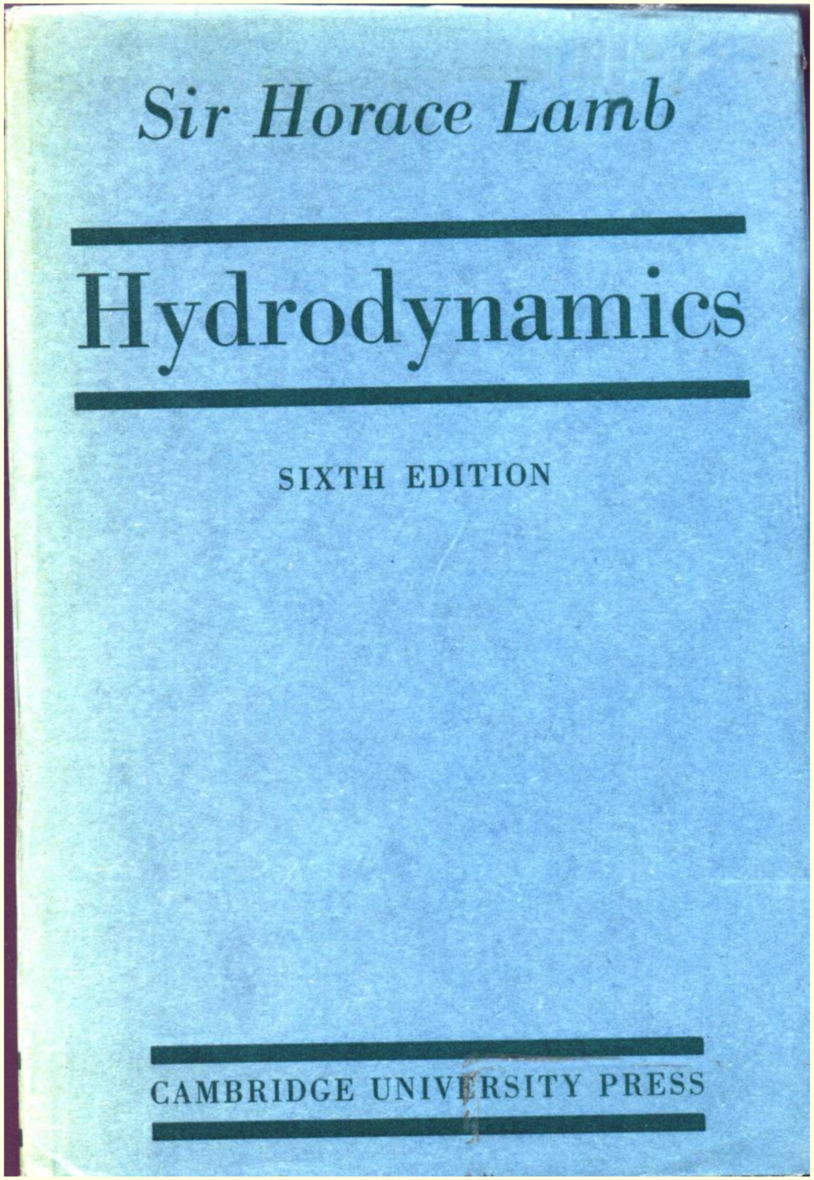
See *Notices of the American Mathematical Society,* Vol 50, 7– 13 (Jan. 2003).

Article on Navier's collapsing bridge.

George G Stokes, 1819–1903

George Gabriel Stokes, founder of modern hydrodynamics.

A crude but indicative metric



In his book *Hydrodynamics*, (6th edition), Horace Lamb has more than 50 page references to Stokes.

Some Contributions of Stokes to Meteorological Science.

- *Stokes' Theorem*
- *Stokes Drag and Stokes' Law*
- *Stokes Drift*
- *Stokes Waves*
- *Campbell-Stokes Sunshine Recorder*
- *Navier-Stokes Equations*

Stokes' Theorem

$$\oint_\Gamma \mathbf{V}\cdot d\mathbf{l} = \iint_\Sigma \nabla \times \mathbf{V} \cdot \mathbf{n}\, da \,.$$

Stokes' Theorem was actually discovered by Kelvin in 1854. It is of central importance in fluid dynamics. It played a rôle in the development of Bjerknes' Circulation Theorem:

$$\frac{dC}{dt} = -\iint_\Sigma \nabla\frac{1}{\rho} \times \nabla p\cdot d\mathbf{a} = -\oint_\Gamma \frac{dp}{\rho}\,,$$

which generalized Kelvin's Circulation Theorem to baroclinic fluids (ρ varying independently of p), and ushered in the study of Geophysical Fluid Dynamics.

Innocent Questions — and —

Innocent Questions — and —

The Victorian Father's Responses

Son: **Pater, why is the sky blue?**
Father: **Son, you must study the works of John Tyndall.**

Innocent Questions — and —

The Victorian Father's Responses

Son: **Pater, why is the sky blue?**
Father: **Son, you must study the works of John Tyndall.**

Son: **Pater, why don't clouds fall down?**
Father: **Son, you must study the works of George Stokes.**

Stokes Drag and Stokes' Law

A more helpful answer:

Son: **Daddy, why don't clouds fall down?**

Dad: **Clouds do fall, but very slowly!**

Stokes Drag and Stokes' Law

A more helpful answer:

Son: **Daddy, why don't clouds fall down?**

Dad: **Clouds do fall, but very slowly!**

Stokes formulated the drag law for small particles in a fluid.

$$F = 6\pi\mu r v$$

This leads an expression for the terminal velocity:

$$v_s = \frac{2r^2\rho g}{9\mu}$$

A particle of radius 5 microns falls with a terminal speed of about 3 mm/s. Thus, it takes about four days to fall through one kilometre.

Stokes' Law was important for Millikan's oil-drop experiment, to measure e/m.

Stokes Flow

Stokes Flow is steady flow in which there is a balance between the viscous and pressure gradient forces:

$$\nu\nabla^2\mathbf{V} = \frac{1}{\rho}\nabla p\,.$$

This balance may be valid for small Reynolds Number.

This balance leads to Stokes' Paradox: Such flow is not possible everywhere. The effect of an obstacle is felt at large distances. Inertial terms are important somewhere.

Hydrodynamics: A study in Logic, Fact and Similitude

by Garrett Birkhoff

Chapter 1 of the book is entitled

HYDRODYNAMICAL PARADOXES.

By a *Paradox*, we mean A plausible argument that yields conclusions at variance with observations.

In fluid systems paradoxes often arise because:

- **Arbitrarily small causes can produce finite effects**
- **An apparent symmetry of causes is not necessarily preserved in the effects**

Some Paradoxes in Hydrodynamics

- **D'Alembert's Paradox**
- **The Reversibility Paradox**
- **Paradoxes of Airfoil Theory**
- **The Rayleigh Paradox**
- **Von Neumann's Paradox**
- **Kopal's Paradox**
- **The Eiffel Paradox**
- **The Rising Bubble Paradox**
- **The Magnus Effect Paradox**
- **Stokes' Paradox**

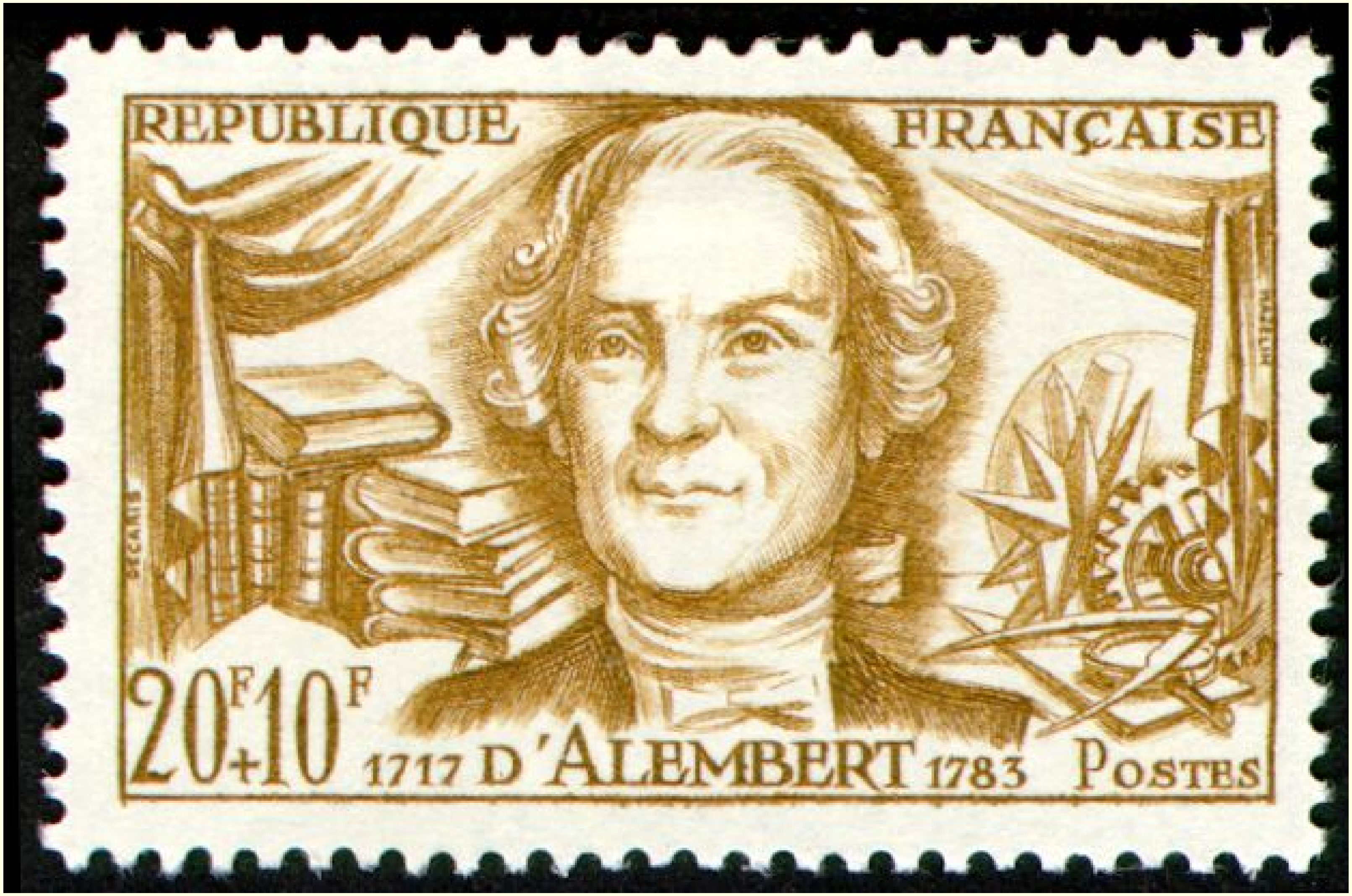
Euler's Equations

Leonhard Euler, born on 15 April, 1707 in Basel. Died on 18 September, 1783 in St Petersburg.
Euler formulated the equations for incompressible, inviscid fluid flow:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \mathbf{g} \,.$$

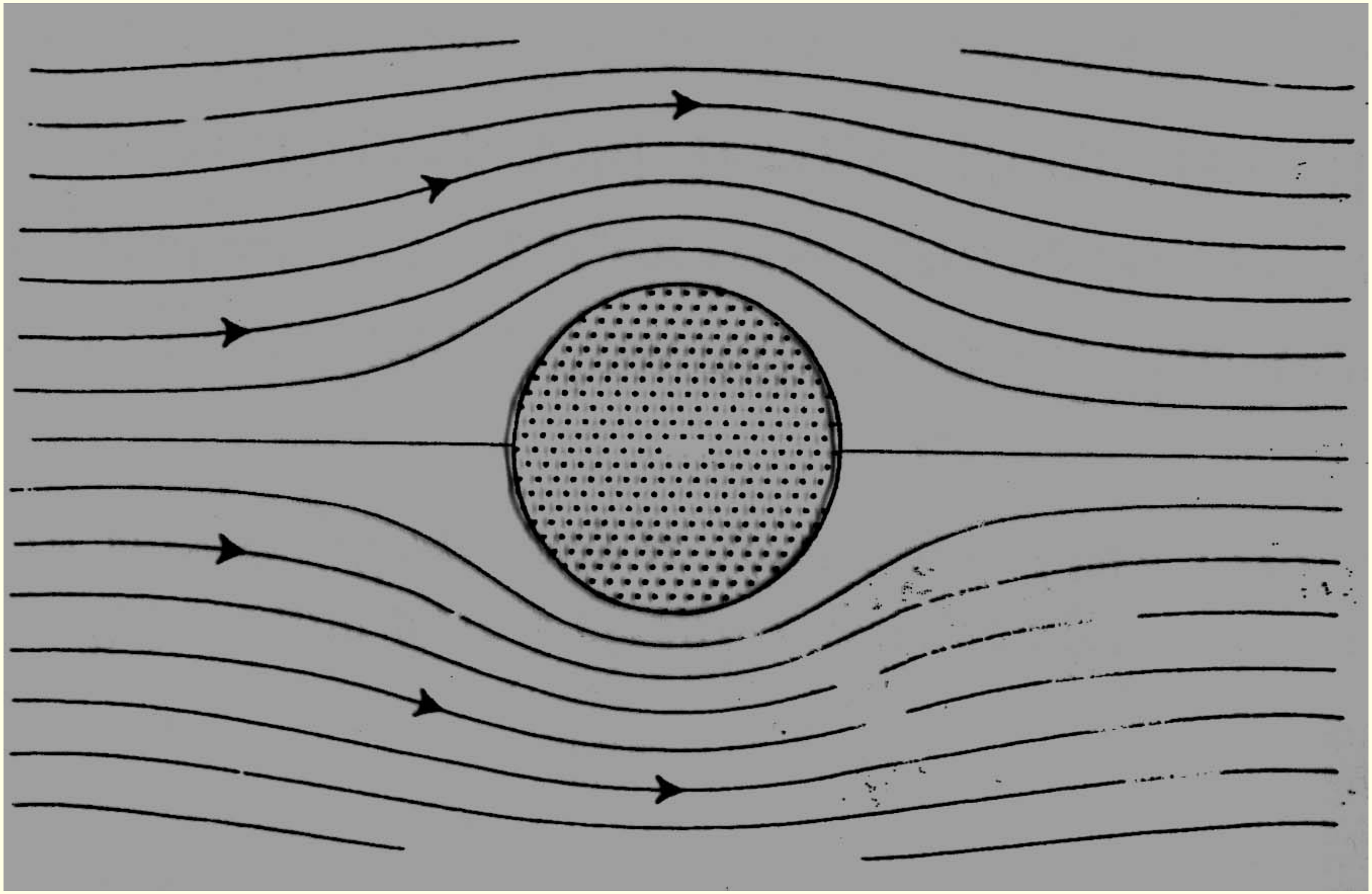
$$\nabla \cdot \mathbf{V} = 0$$

Jean Le Rond d'Alembert



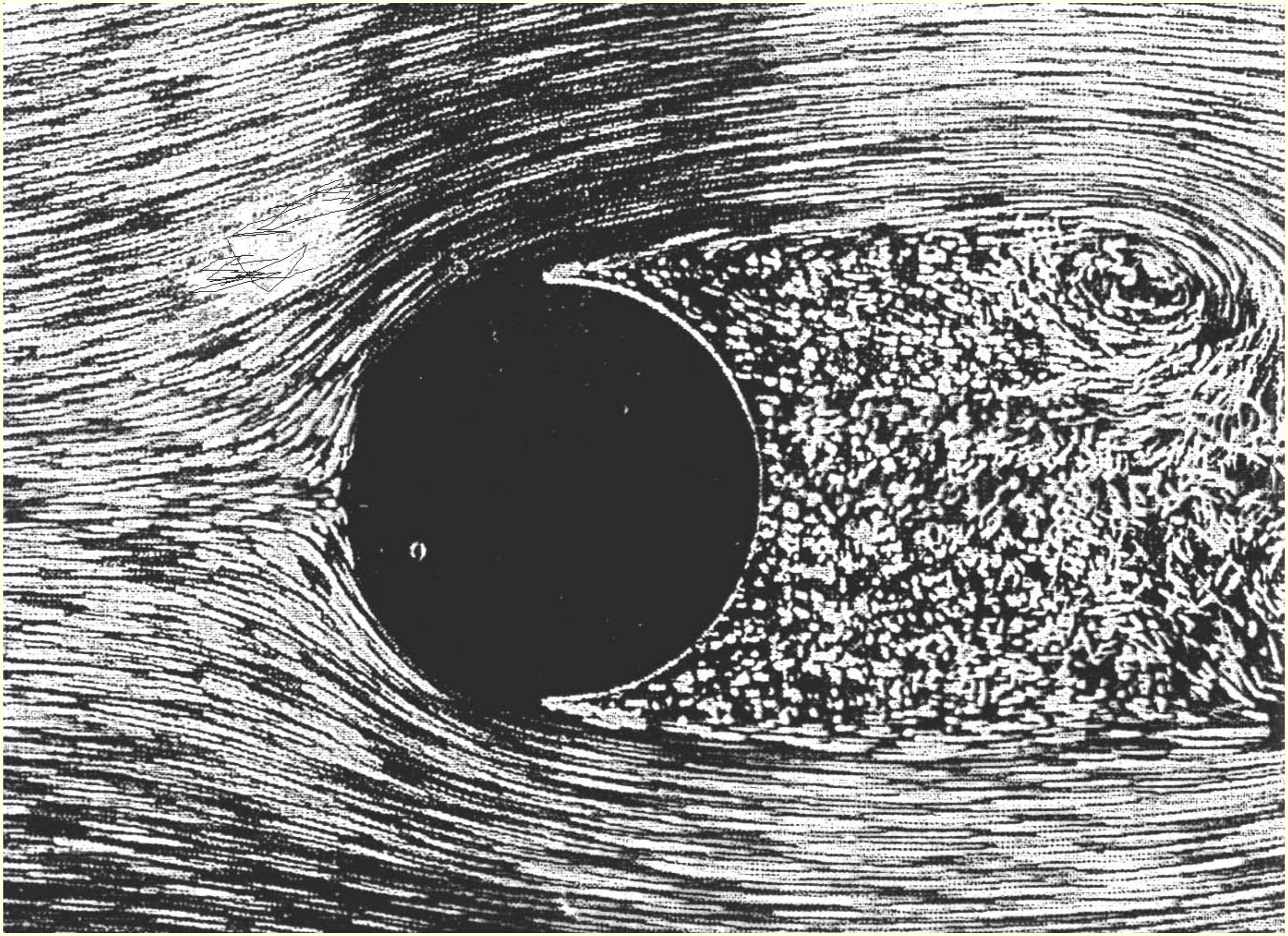
A body moving at constant speed through a gas or a fluid does not experience any resistance (D'Alembert 1752).

Hypothetical Fluid Flow



Purely Inviscid Flow. Upstream-downstream symmetry.

Actual Fluid Flow



Viscous Flow. Strong upstream-downstream assymmetry.

Resolution of d'Alembert's Paradox

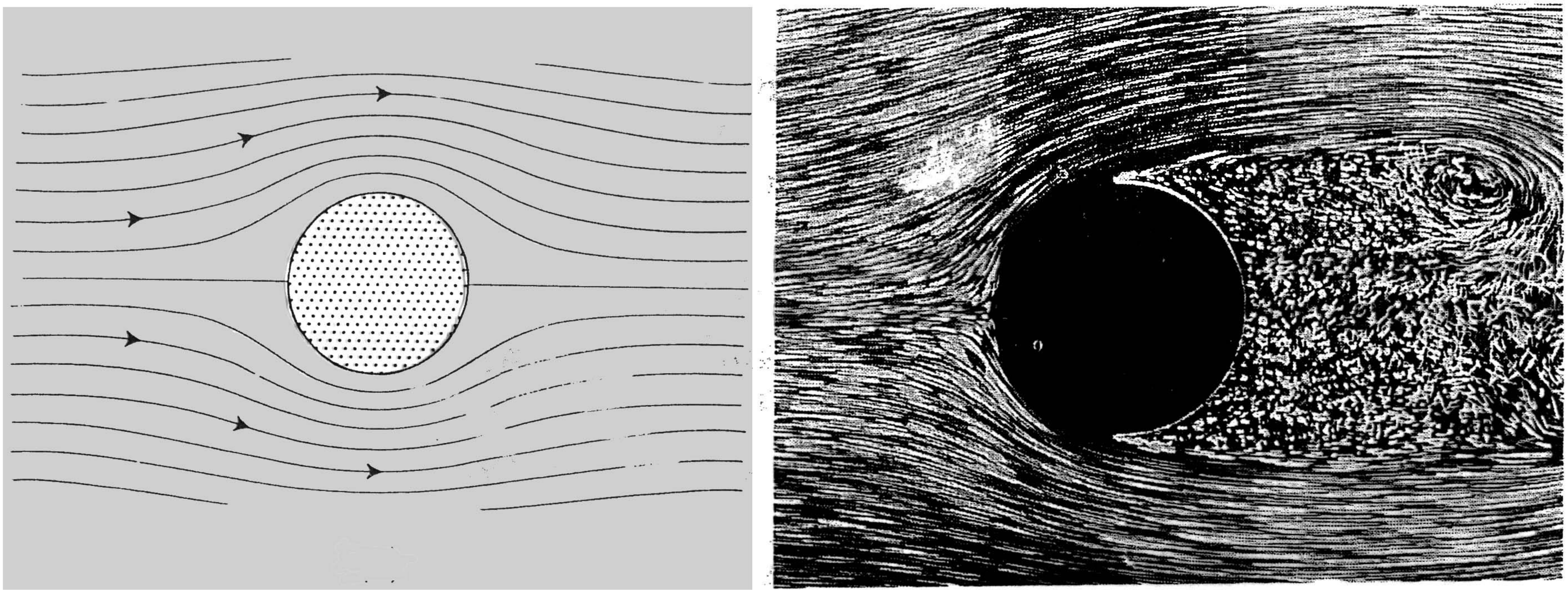


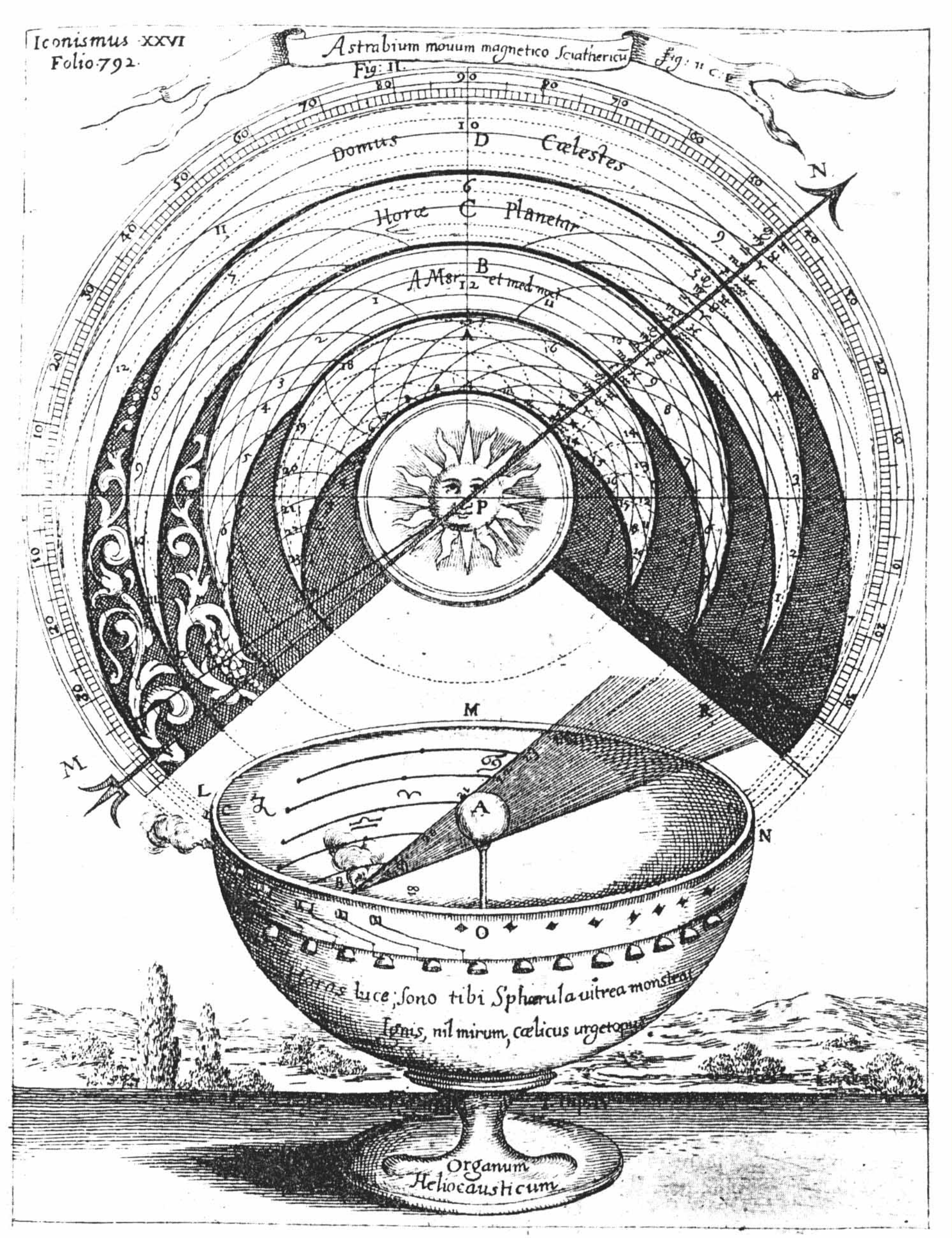
Fig. 9.1 Flow past a circular cylinder for (a) a hypothetical fluid with zero viscosity, (b) a real fluid with very small viscosity μ *(from van Dyke 1982).*

The minutest amount of viscosity has a profound qualitative impact on the character of the solution.

The Navier-Stokes equations incorporate the effect of viscosity.

The Campbell-Stokes Sunshine Recorder

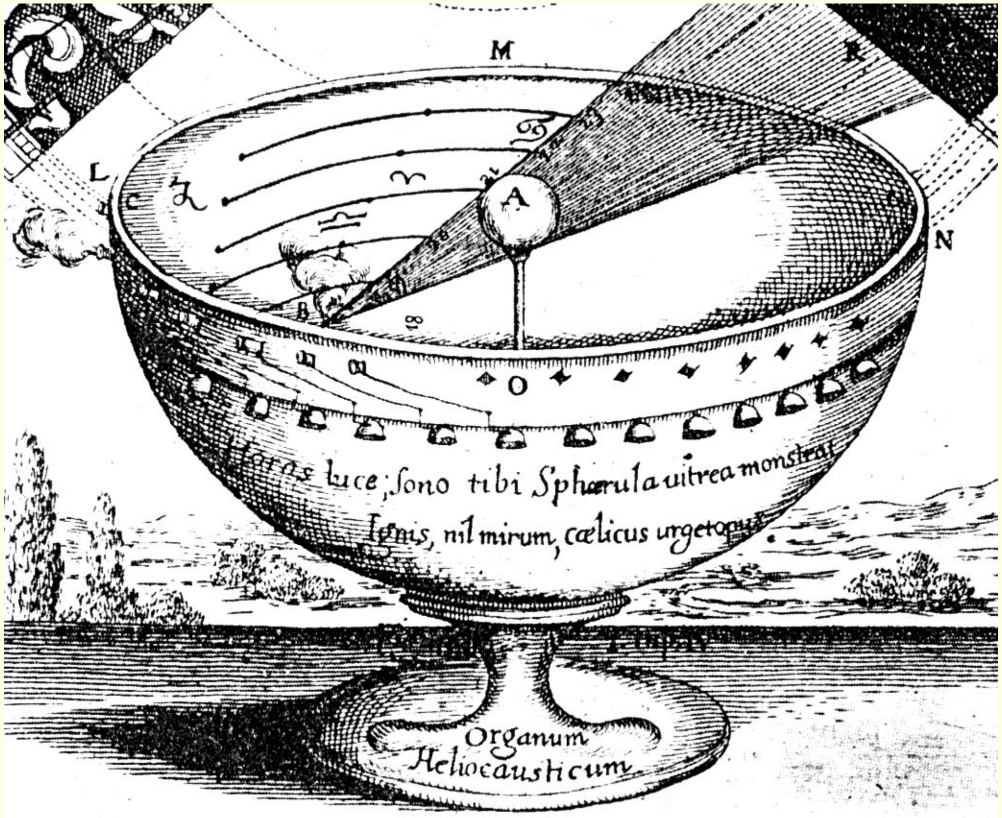
An Early Sunshine Recorder



Athanasius Kircher was Professor of Mathematics and Hebrew at the *Collegio Romano*. Around 1646 he devised a recording sundial called the **Horologium Heliocausticum.**

The Horologium Helio-causticum

A Sundial is drawn in the shell, "together with things for burning and making sounds."



With Light and sound the glassy sphere shows thee the hours; Truly, it is the work of the heavenly fire.

Campbell's Sunshine Recorder.

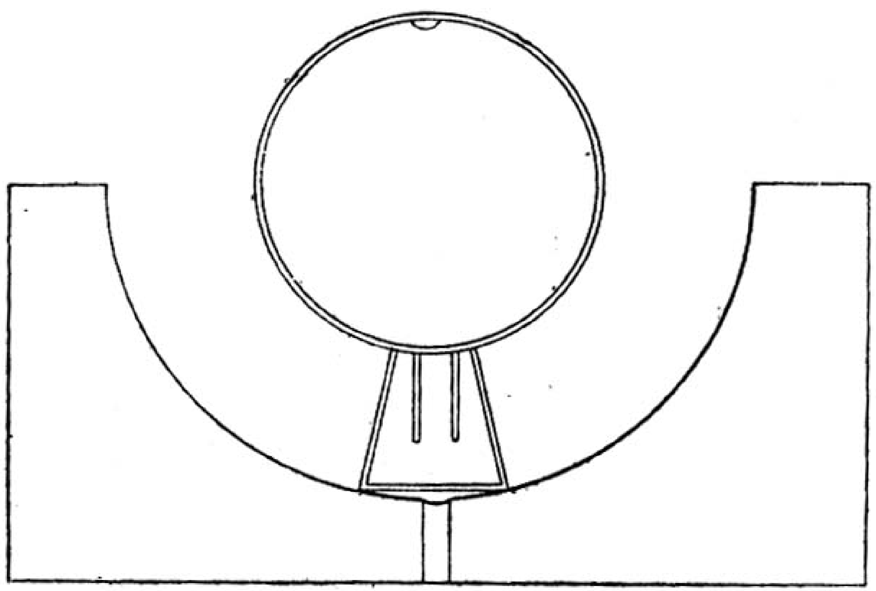


FIG. 1.—Section of Mr. Campbell's original Sunshine Recorder.

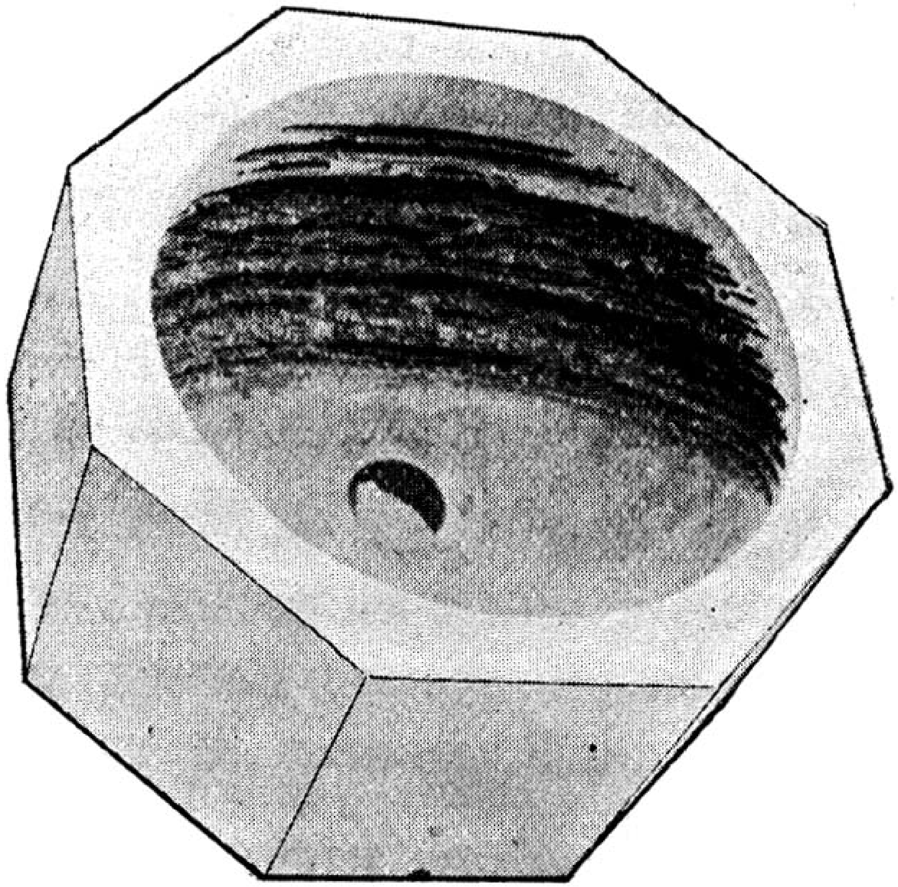
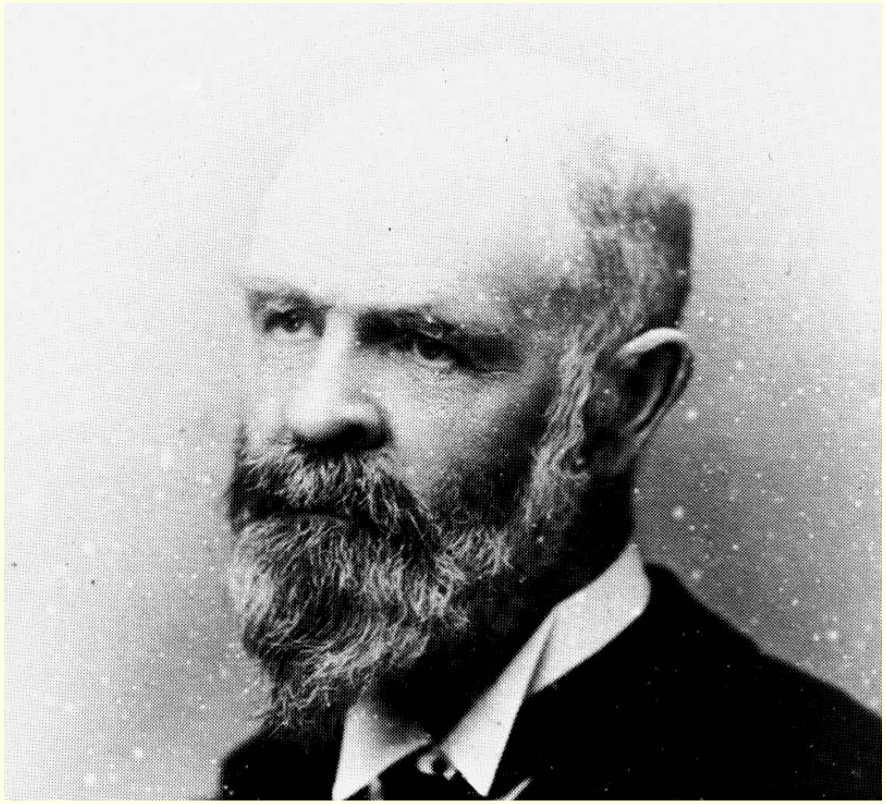


FIG. 2.—Wooden Sunshine Bowl.

The "self-registering sundial" of J. F. Campbell (c. 1853).

Robert Henry Scott (1833–1916)



Robert Scott, born in Dublin, was founder of Valentia Observatory and first Director of the British Meteorological Office.

Scott proposed some improvements to Campbell's sunshine recorder.

The detailed design of the instrument was due to Stokes.

Stokes' Quarterly Journal Paper

Description of the Card Supporter for Sunshine Recorders adopted at the Meteorological Office

George Gabriel Stokes

Quarterly Journal of the Royal Meteorological Society, **Vol. 6 (1880) 83–94.**

"The method of recording sunshine by the burning of an object placed in the focus of a glass sphere freely exposed to the rays of the sun, which was devised by Mr. Campbell, commends itself by its simplicity, and seems likely to come into pretty general use."

Stokes' Quarterly Journal Paper

Description of the Card Supporter for Sunshine Recorders adopted at the Meteorological Office

George Gabriel Stokes

Quarterly Journal of the Royal Meteorological Society, **Vol. 6 (1880) 83–94.**

"The method of recording sunshine by the burning of an object placed in the focus of a glass sphere freely exposed to the rays of the sun, which was devised by Mr. Campbell, commends itself by its simplicity, and seems likely to come into pretty general use."

In the discussion following the reading of the paper, a Mr. Mawley remarked:

"The fact of this sunshine-recorder being in all respects an English invention, adds much to its interest."

Campbell-Stokes Sunshine Recorder

FIGURE 138
Campbell-Stokes sunshine recorder.

One moving part!
(In Biblical Coordinates)

The Navier-Stokes Equations

Navier, C. L. M. H., 1822: Mémoire sur les lois du mouvement des fluides.
Mém. Acad. Sci. Inst. France, **Vol. 6, 389–440.**

Stokes, G. G., 1845: On the theories of the internal friction of fluids in motion.
Trans. Cambridge Philos. Soc., **Vol. 8.**

Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g}^{\star} .$$

The Navier-Stokes Equations describe how the change of velocity, the acceleration of the fluid, is determined by the pressure gradient force, the gravitational force and the frictional force.

Navier-Stokes Equations

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g}^\star \,.$$

The Navier-Stokes Equations describe how the change of velocity, the acceleration of the fluid, is determined by the pressure gradient force, the gravitational force and the frictional force.

For motion relative to the rotating earth, we must include the Coriolis force:

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\boldsymbol{\Omega} \times \mathbf{V} + \frac{1}{\rho} \nabla p = \nu \nabla^2 \mathbf{V} + \mathbf{g} \,.$$

Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)

Relates the pressure, temperature and density

CONTINUITY EQUATION

Conservation of mass; air neither created nor distroyed

WATER CONTINUITY EQUATION

Conservation of water (liquid, solid and gas)

HYDROSTATIC LAW

Balance between gravity and vertical pressure gradient

EQUATIONS OF MOTION: Navier-Stokes Equations

Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction

Six equations; Seven variables (u, v, w, ρ, p, T, q).

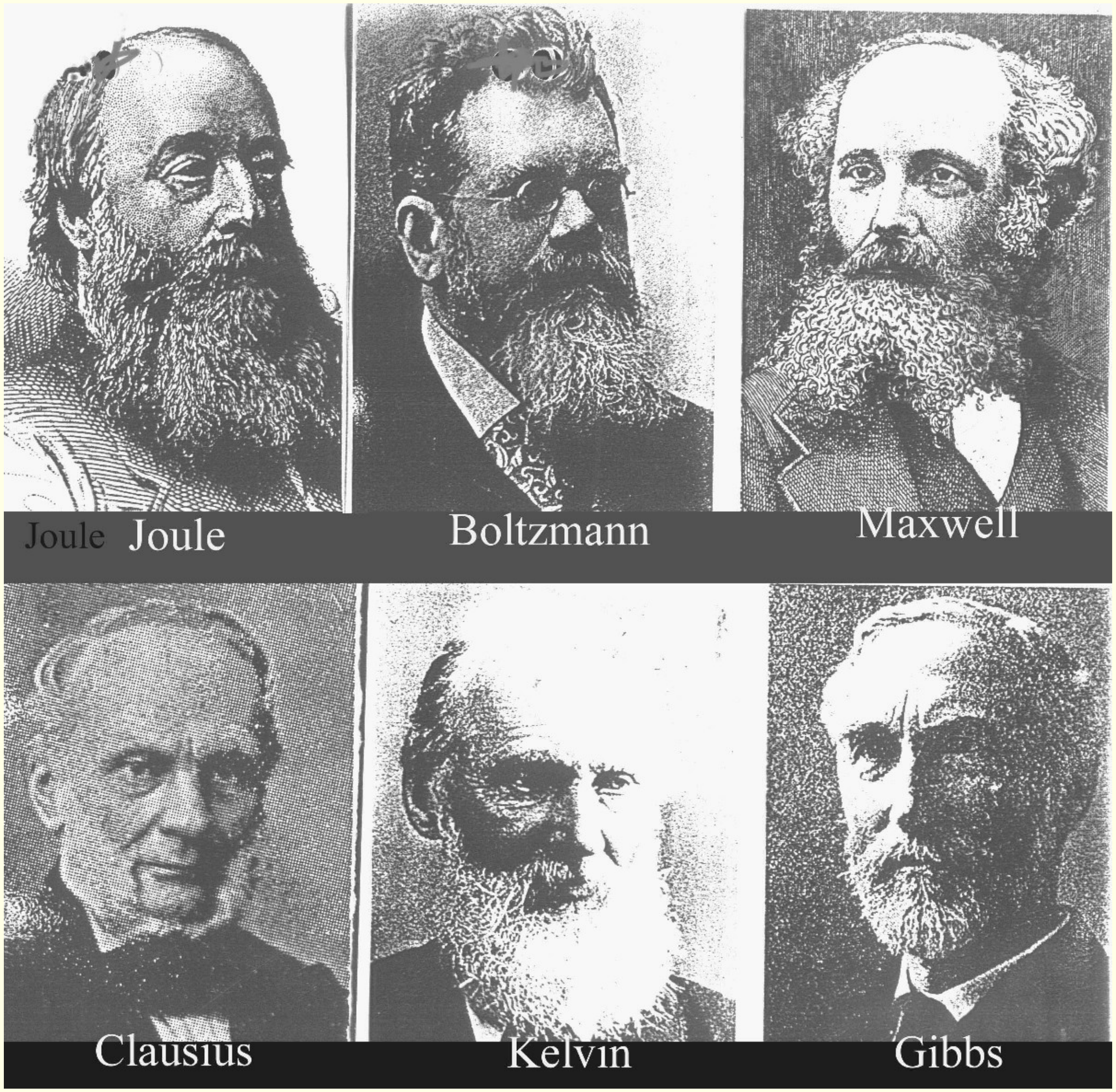
Equations of the Atmosphere

GAS LAW (Boyle's Law and Charles' Law.)
Relates the pressure, temperature and density
CONTINUITY EQUATION
Conservation of mass; air neither created nor distroyed
WATER CONTINUITY EQUATION
Conservation of water (liquid, solid and gas)
HYDROSTATIC LAW
Balance between gravity and vertical pressure gradient
EQUATIONS OF MOTION: Navier-Stokes Equations
Describe how the change of velocity is determined by the pressure gradient, Coriolis force and friction

Six equations; Seven variables (u, v, w, ρ, p, T, q).

THERMODYNAMIC EQUATION
Determines changes of temperature due to heating or cooling, compression or rarifaction, etc.

The Hairy Men of Thermo-D



It would appear from this sample that a fulsome beard may serve as a thermometer of proficiency in thermodynamics.

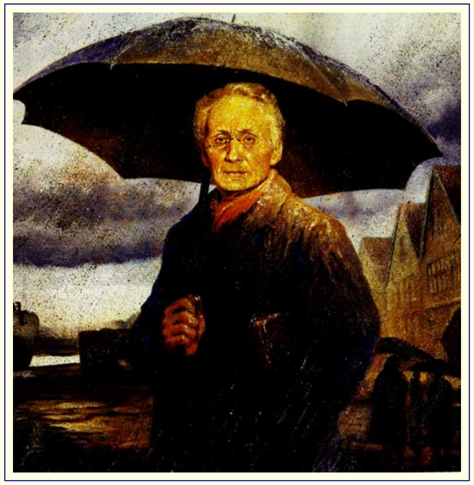
However, more exhaustive research is required before a definitive conclusion can be reached.

Scientific Weather Forecasting in a Nut-Shell

- **The atmosphere is a physical system**
- **Its behaviour is governed by the laws of physics**
- **These laws are expressed quantitatively in the form of mathematical equations**
- **Using observations, we can specify the atmospheric state at a given initial time: "Today's Weather"**
- **Using the equations, we can calculate how this state will change over time: "Tomorrow's Weather"**
- **The equations are very complicated (non-linear) and a powerful computer is required to do the calculations**
- **The accuracy decreases as the range increases; there is an inherent limit of predictibility.**

Vilhelm Bjerknes (1862–1951)

Vilhelm Bjerknes (1862–1951)



Vilhelm Bjerknes

- **Born March, 1862.**
- **Matriculated in 1880.**
- **Fritjøf Nansen was a fellow-student.**
- **Paris, 1989–90. Studied under Poincare.**
- **Bonn, 1890–92. Worked with Heinrich Hertz.**
- **Stockholm, 1983–1907.**

– **1898: Circulation theorems**
  – **1904: Meteorological Manifesto**

- **Christiania (Oslo), 1907–1912.**
- **Leipzig, 1913–1917.**
- **Bergen, 1917–1926.**
  – **1919: Frontal Cyclone Model.**
- **Oslo, 1926 — (retired 1937).**
  - **Died, April 9,1951.**

Bjerknes' 1904 Manifesto

To establish a science of meteorology, with the central aim of predicting future states of the atmosphere from the present state.

"If it is true, as every scientist believes, that subsequent atmospheric states develop from the preceeding ones according to physical law, then it is apparent that the necessary and sufficient conditions for the rational solution of forecasting problems are the following:

1. **A sufficiently accurate knowledge of the state of the atmosphere at the initial time**
2. **A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another."**

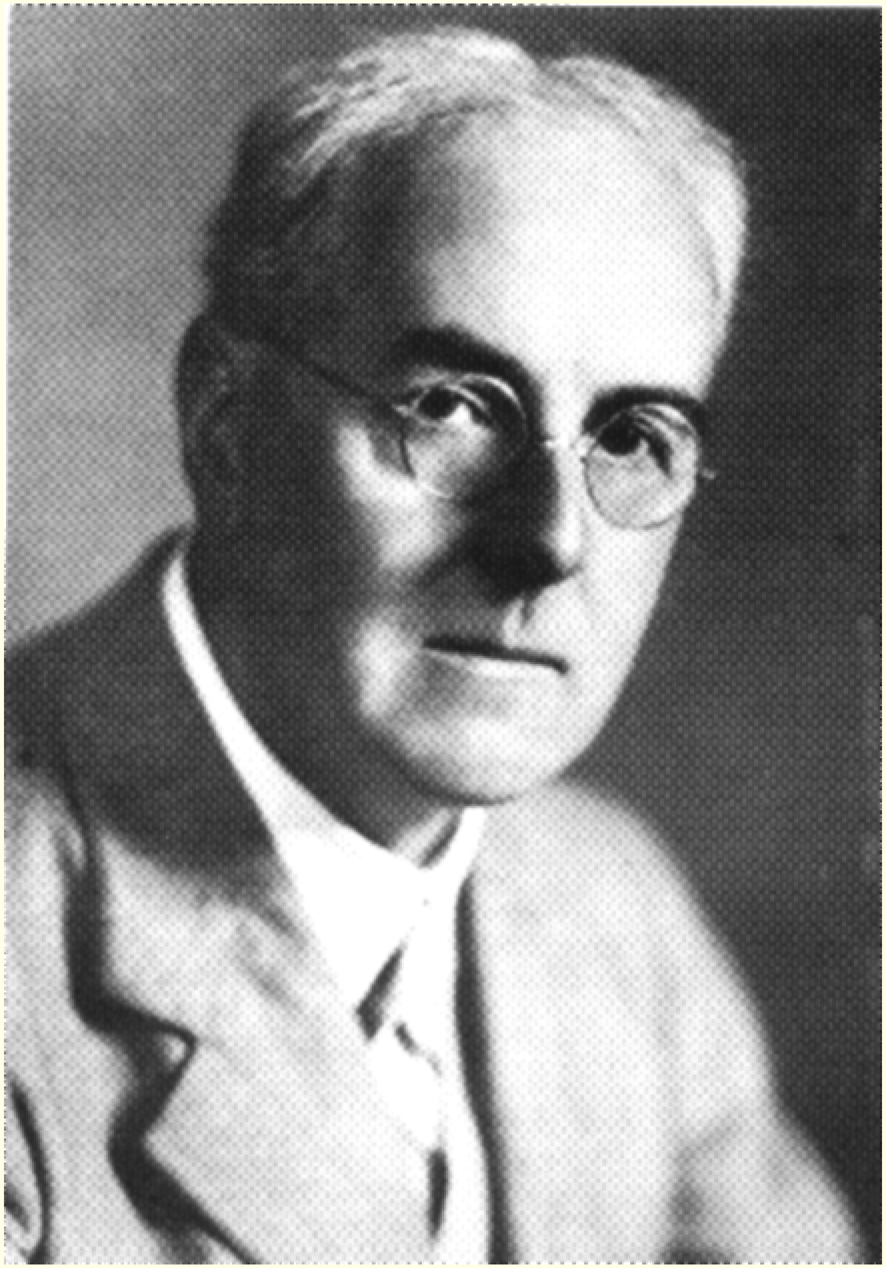
Step (1) is Diagnostic. **Step (2) is Prognostic.**

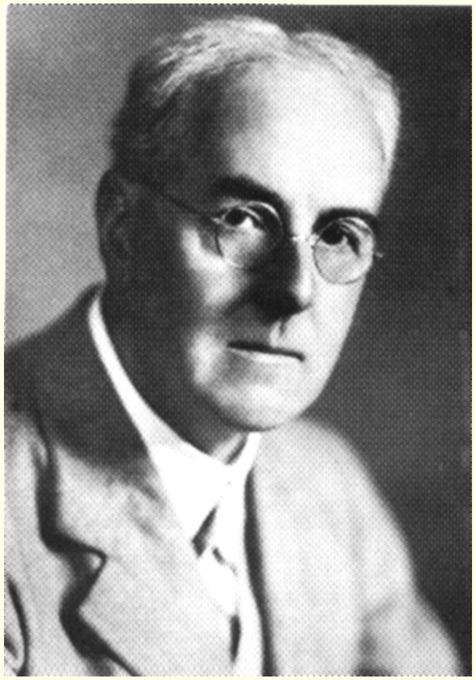
Bjerknes ruled out analytical solution of the mathematical equations, due to their nonlinearity and complexity:

"For the solution of the problem in this form, graphical or mixed graphical and numerical methods are appropriate, which methods must be derived either from the partial differential equations or from the dynamical-physical principles which are the basis of these equations."

However, there was a scientist more bold — or foolhardy — than Bjerknes, who actually tried to calculate future weather. This was Lewis Fry Richardson

Lewis Fry Richardson, 1881–1953.

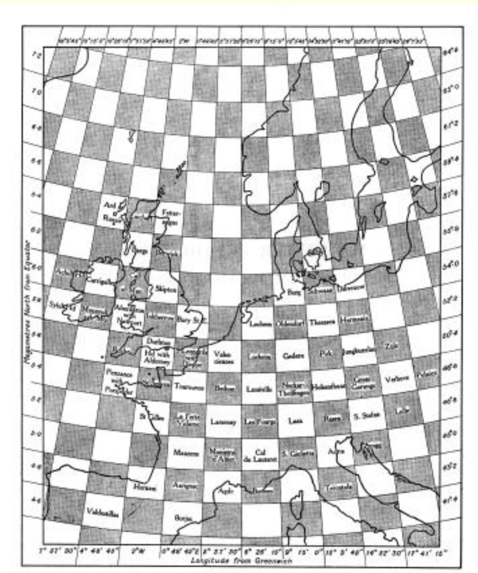




- Born, 11 October, 1881, Newcastle-upon-Tyne
- Family background: well-known quaker family
- 1900–1904: Kings College, Cambridge
- 1913–1916: Met. Office. Superintendent, Eskdalemuir Observatory
- Resigned from Met Office in May, 1916. Joined Friends' Ambulance Unit.
- 1919: Re-employed by Met. Office
- 1920: M.O. linked to the Air Ministry. LFR Resigned, on grounds of concience
- 1922: *Weather Prediction by Numerical Process*
- 1926: Break with Meteorology. Worked on Psychometric Studies. Later on Mathematical causes of Warfare
- 1940: Resigned to pursue "peace studies"
- Died, September, 1953.

Richardson contributed to Meteorology, Numerical Analysis, Fractals, Psychology and Conflict Resolution.

The Finite Difference Scheme



The globe is divided into cells, like the checkers of a chess-board.

Spatial derivatives are replaced by finite differences:

$$\frac{df}{dx} \to \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}.$$

Similarly for time derivatives:

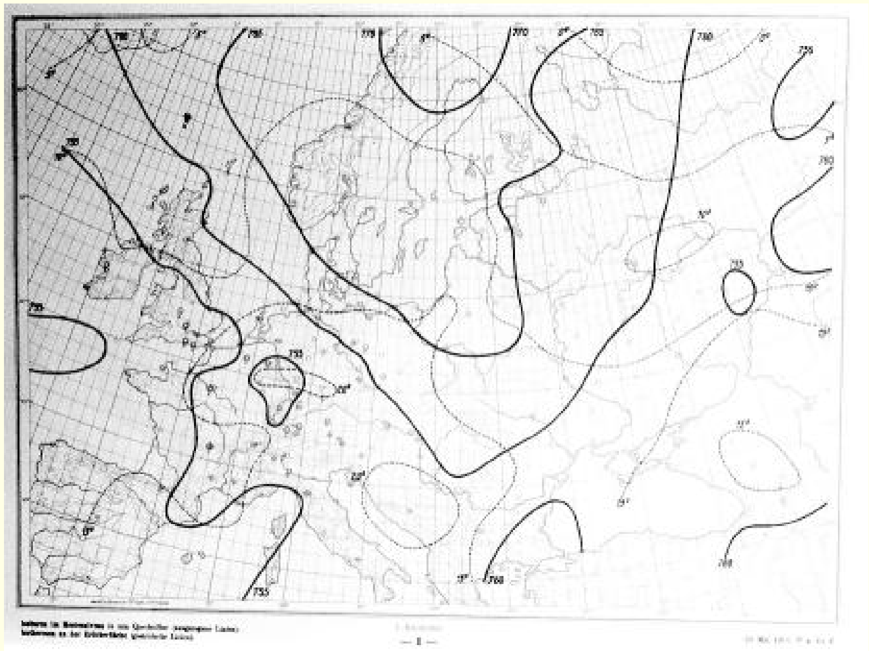
$$\frac{dQ}{dt} \to \frac{Q^{n+1} - Q^{n-1}}{2\Delta t} = F^n$$

This is immediately solved for Q^{n+1}:

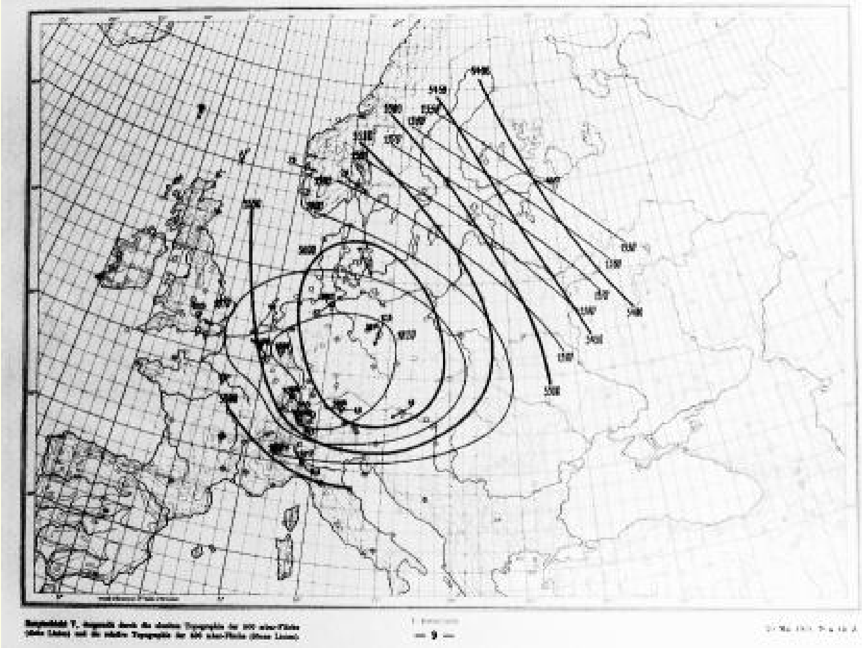
$$Q^{n+1} = Q^{n-1} + 2\Delta t F^n .$$

By repeating the calculations for many time steps, we can get a forecast of any length. Richardson calculated only the initial rates of change.

The Leipzig Charts for 0700 UTC, May 20, 1910



Bjerknes' sea level pressure analysis.



Bjerknes' 500 hPa height analysis.

Richardson's *Spread-sheet*

COMPUTING FORM P XIII. Divergence of horizontal momentum-per-area. Increase of pressure

The equation is typified by: $-\frac{\partial R_{86}}{\partial t} = \frac{\partial M_{E86}}{\partial e} + \frac{\partial M_{N86}}{\partial n} - M_{N86}\frac{\tan\phi}{a} + m_{H6} - m_{H8}{}^{*} + \frac{2}{a}M_{H86}$. (See Ch. 4/2#5.)

* In the equation for the lowest stratum the corresponding term $-m_{G8}$ does not appear

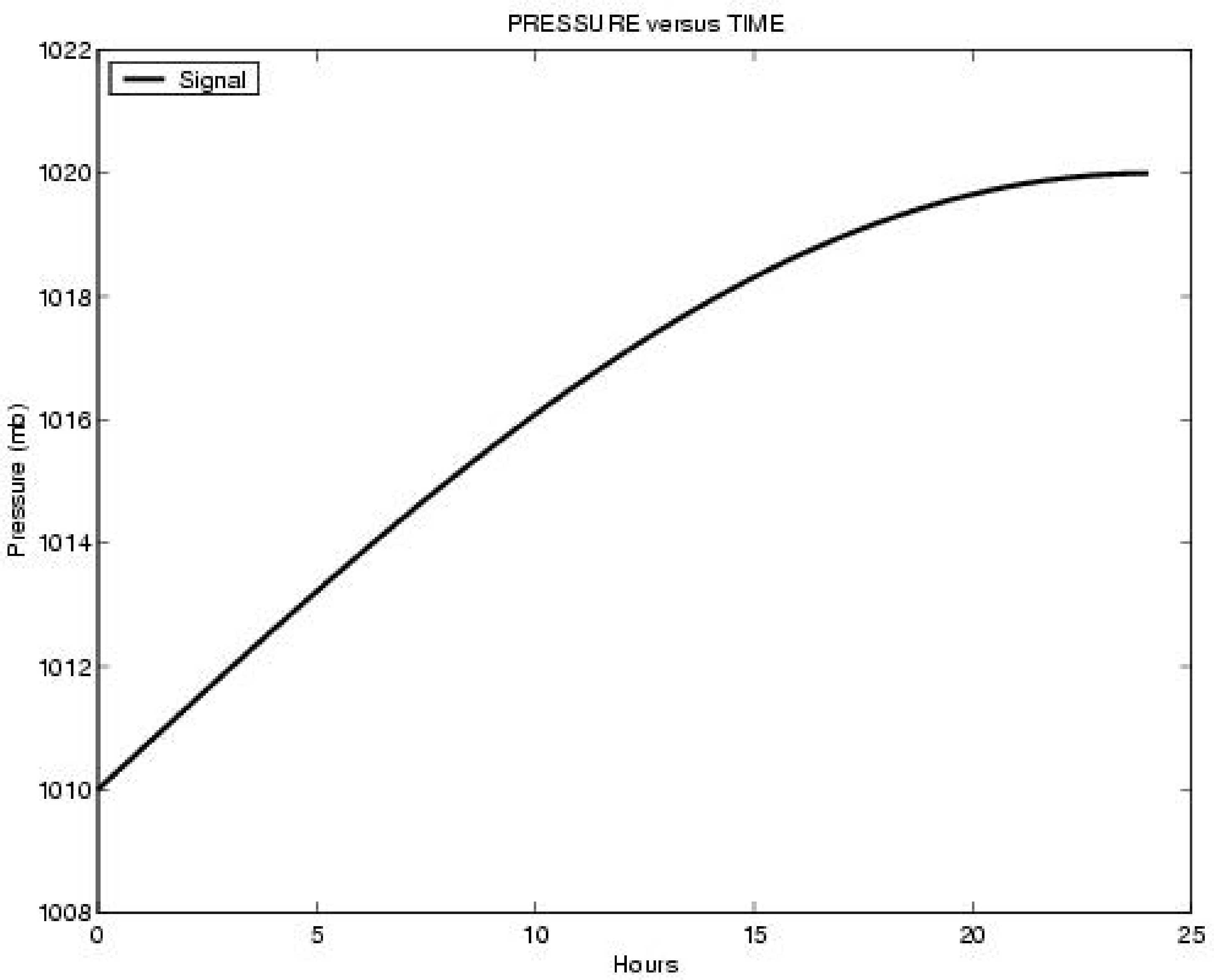
Longitude *11° East* $\delta e = 441 \times 10^5$ | Latitude *5400 km North* $\delta n = 400 \times 10^5$ | Instant *1910 May 20ᵈ 7ʰ G.M.T.* $a^{-1}.\tan\phi = 1{\cdot}78 \times 10^{-9}$ | Interval, δt *6 hours* $a = 6{\cdot}36 \times 10^8$

REF.:—				previous 3 columns	previous column	Leave the subsequent columns to be filled up after the vertical velocity has been computed on Form P XVI	Form P XVI	Form P XVI	equation above	previous column	previous column	previous column
h	$\frac{\delta M_E}{\delta e}$	$\frac{\delta M_N}{\delta n}$	$-\frac{M_N \tan\phi}{a}$	$\mathrm{div}'_{EN}M$	$-g\delta t\,\mathrm{div}'_{EN}M$		m_H	$\frac{2M_H}{a}$	$-\frac{\partial R}{\partial t}$	$+\frac{\partial R}{\partial t}\delta t$	$g\frac{\partial R}{\partial t}\delta t$	$\frac{\partial p}{\partial t}\delta t$
	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$	$100\times$		$10^{-5}\times$	$10^{-5}\times$	$10^{-5}\times$		$100\times$	$100\times$
h_0							0					0
	−61	−245	−6	−312	656				−229	49·5	483	
h_2							−83					483
	367	−257	2	112	−236			0·06	−136	29·4	287	
h_4							165					770
	93	−303	−16	−226	478			0·11	−124	26·8	262	
h_6							63					1032
	32	− 55	−12	− 35	74			0·07	−110	23·8	233	
h_8							138					1265
	−256	38	− 8	−226	479			0·03	− 88	19·0	186	
h_G												1451
NOTE: $\mathrm{div}'_{EN}M$ is a contraction for $\frac{\delta M_E}{\delta e} + \frac{\delta M_N}{\delta n} - M_N\frac{\tan\phi}{a}$					SUM = *1451* $= \frac{\partial p_G}{\partial t}\delta t$							check by $\Sigma - g\delta t\,\mathrm{div}'_{EN}M$

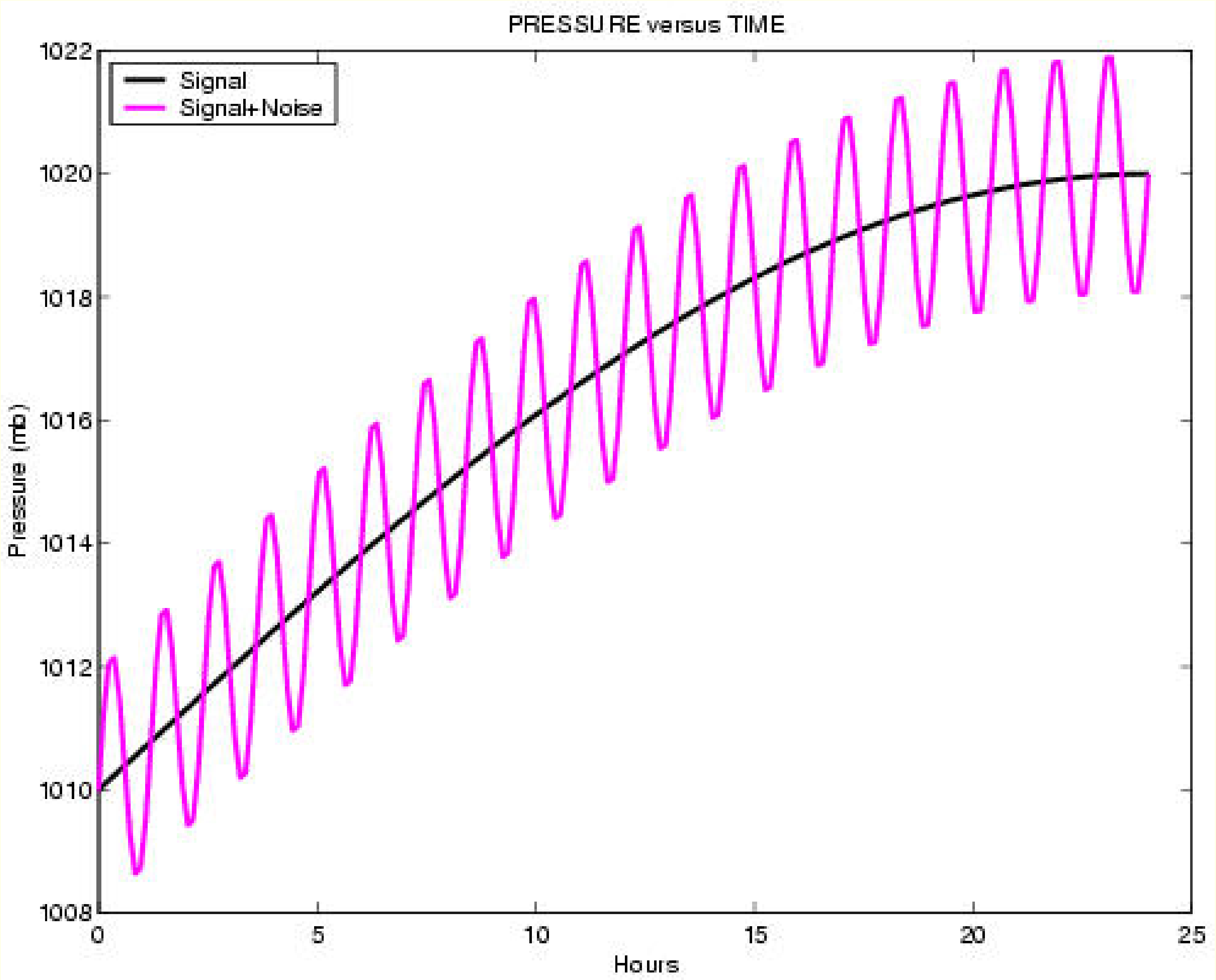
Richardson's Computing Form $\mathrm{P}_{\mathrm{XIII}}$

The figure in the bottom right corner is the forecast change in surface pressure: 145 mb in six hours!

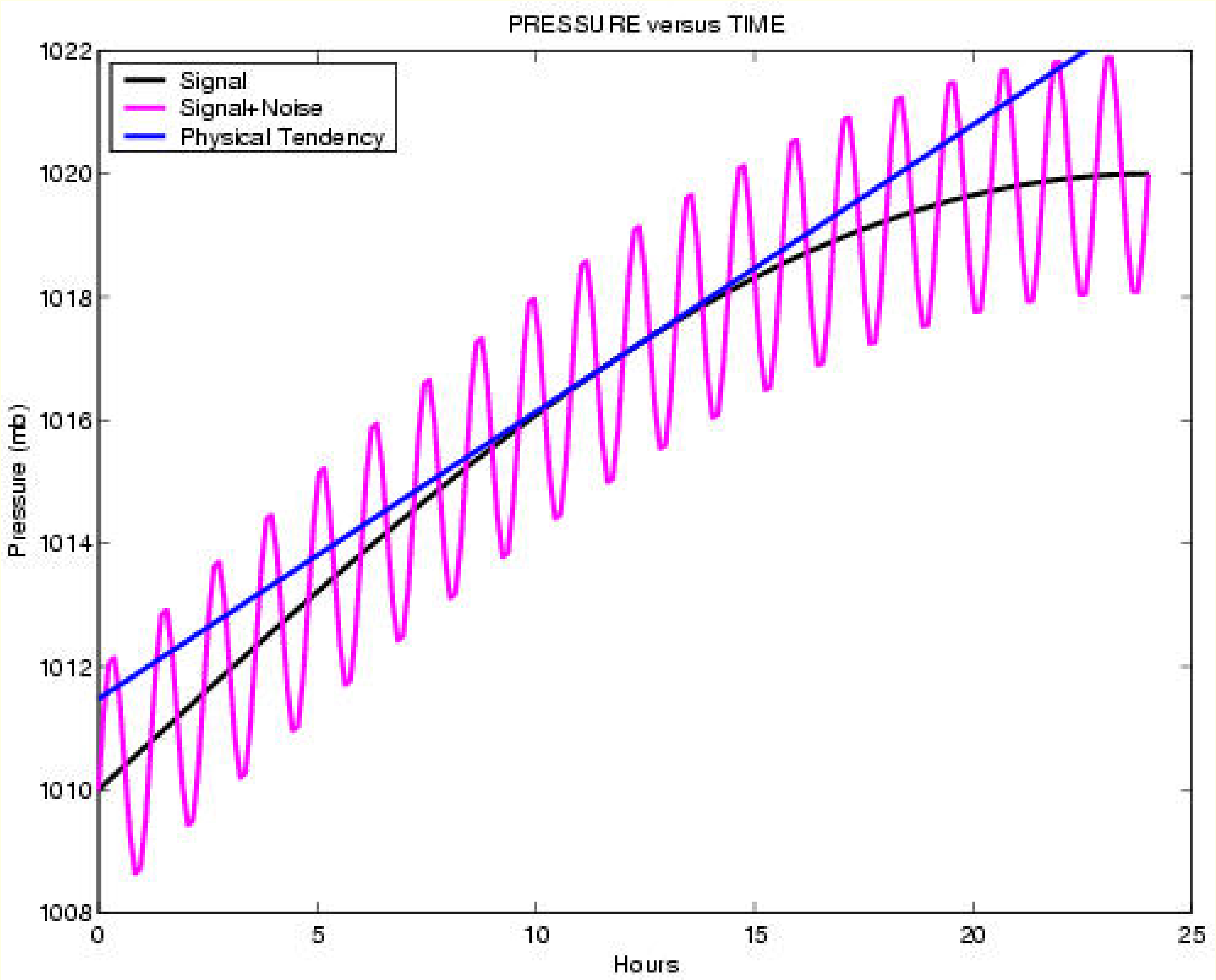
Smooth Evolution of Pressure



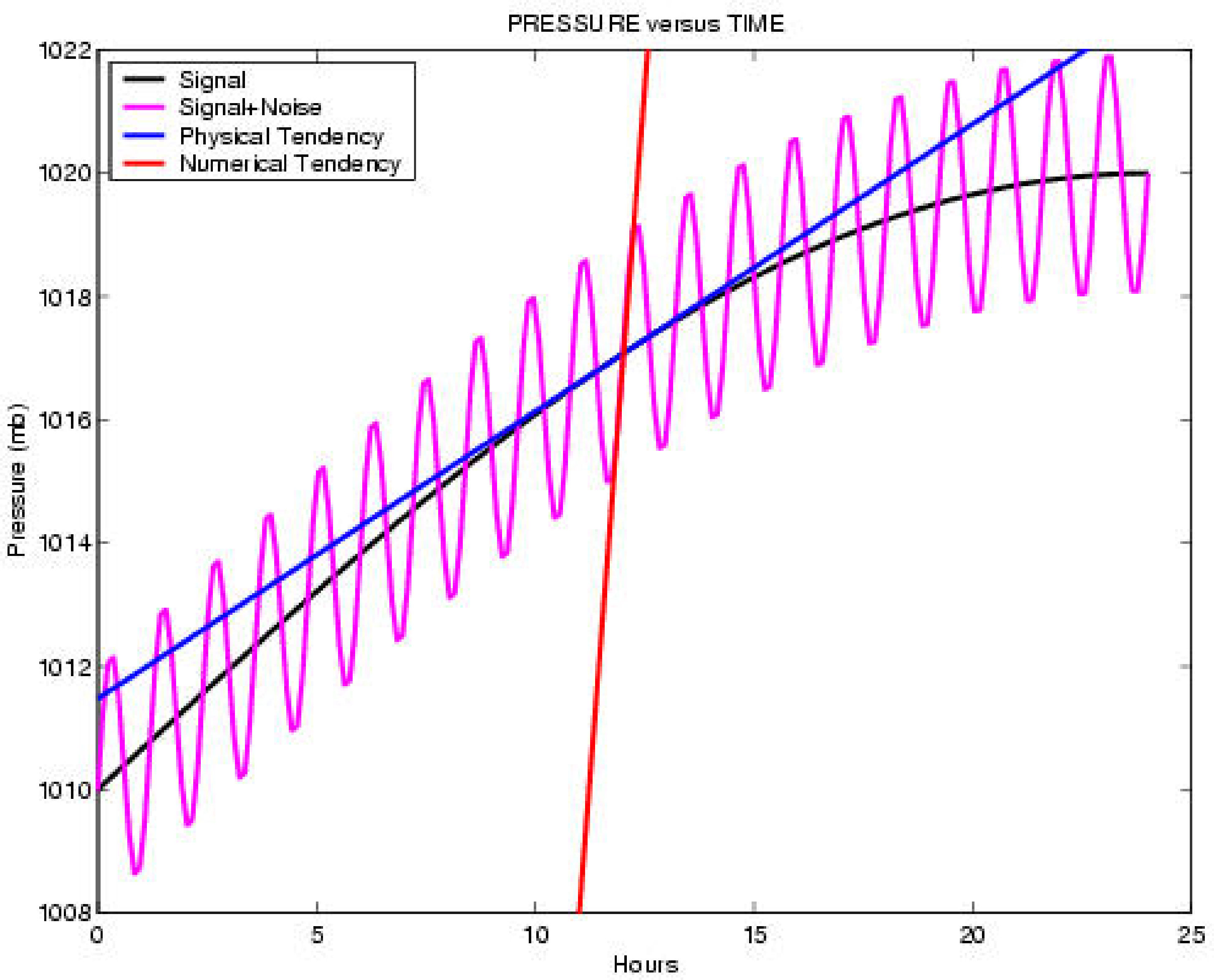
Noisy Evolution of Pressure

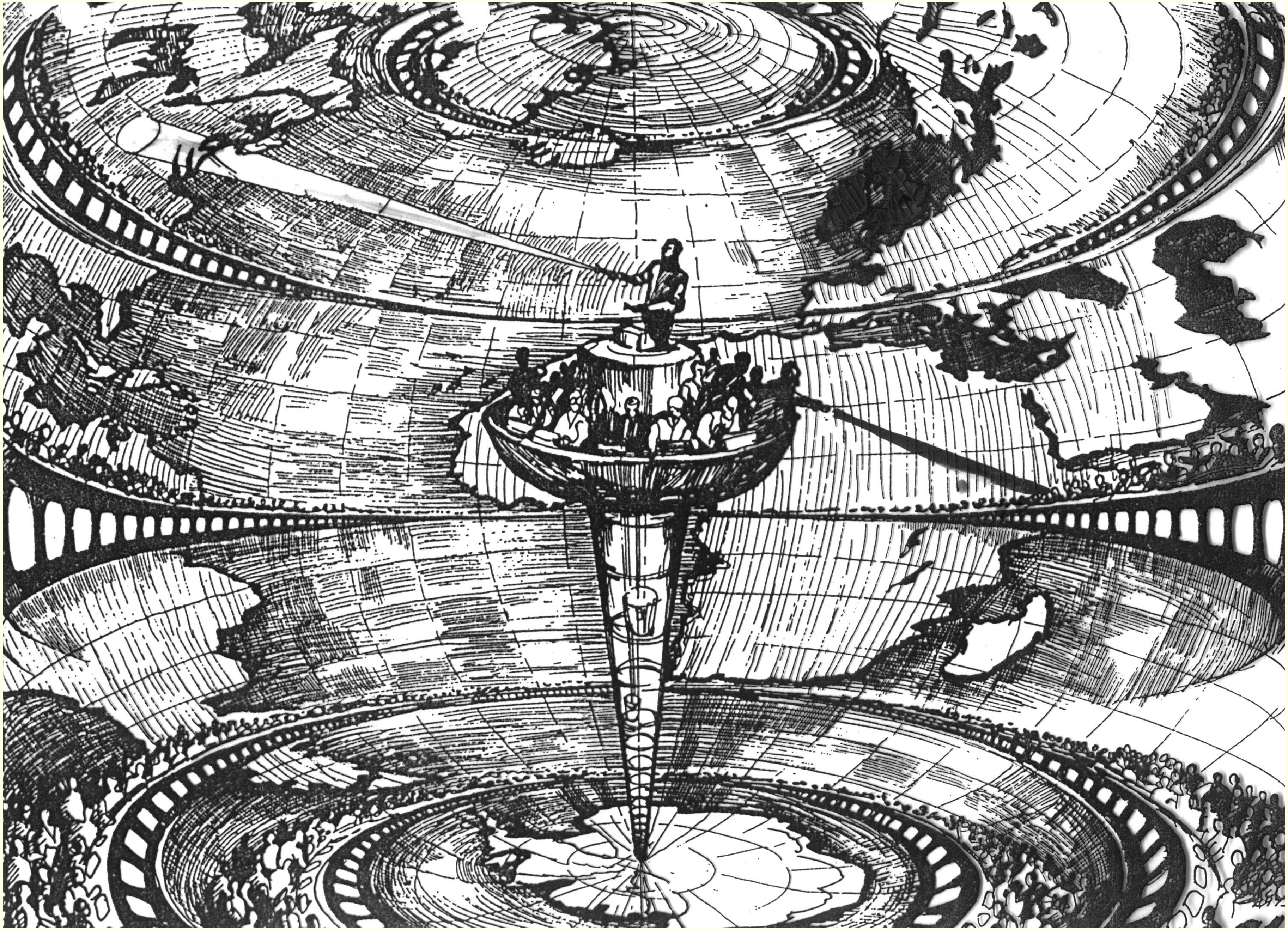


Tendency of a Smooth Signal



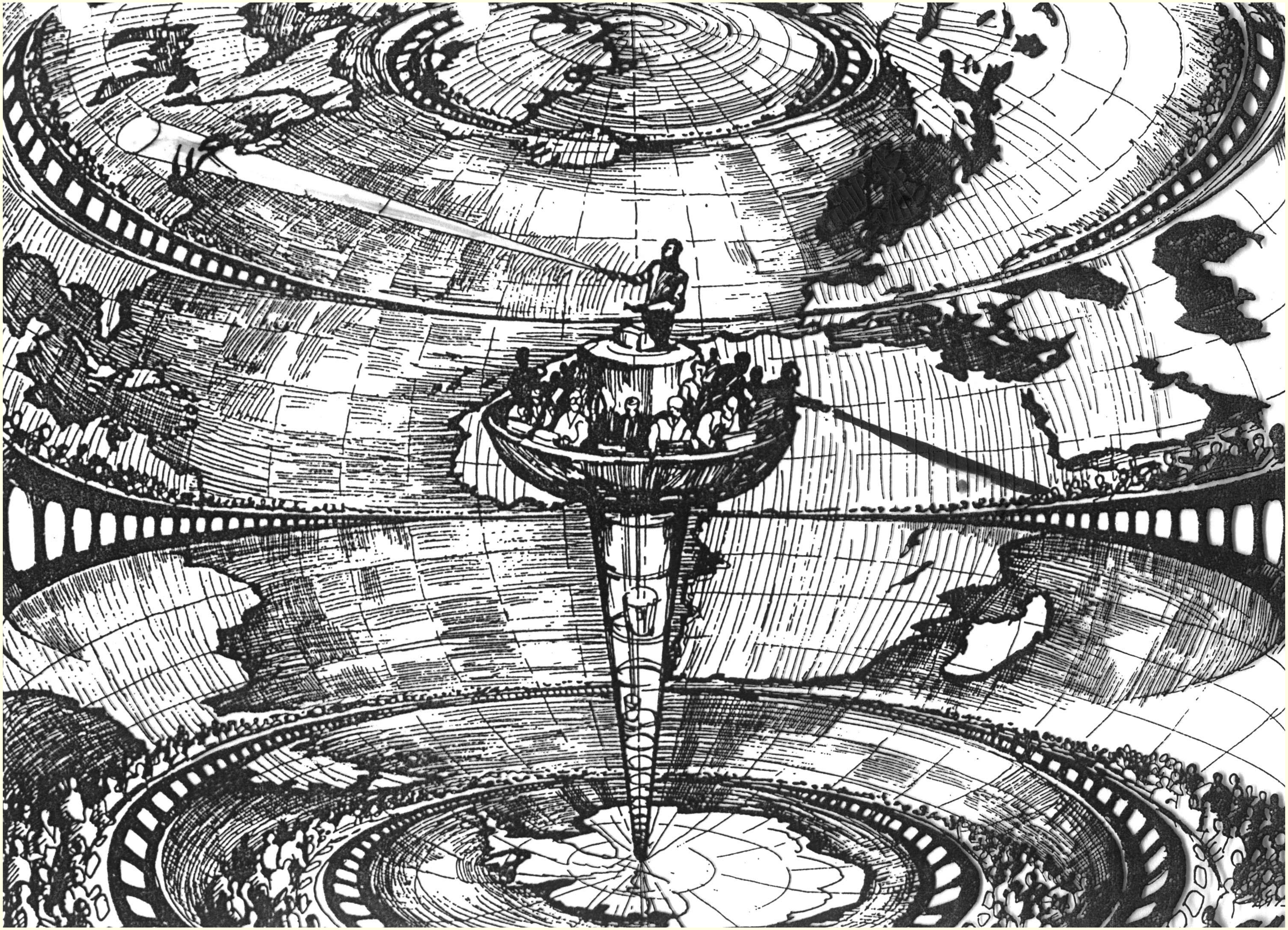
Tendency of a Noisy Signal





Richardson's Forecast Factory (A. Lannerback).

Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984



Richardson's Forecast Factory (A. Lannerback).

Dagens Nyheter, Stockholm. Reproduced from L. Bengtsson, *ECMWF*, 1984

64,000 Computers: The first Massively Parallel Processor

Advances 1920–1950

- ***Dynamic Meteorology***
  - Rossby Waves
  - Quasi-geostrophic Theory
  - Baroclinic Instability
- ***Numerical Analysis***
  - CFL Criterion
- ***Atmopsheric Observations***
  - Radiosonde
- ***Electronic Computing***
  - ENIAC

The ENIAC

Electronic Computer Project, 1946 (under direction of John von Neumann)

Von Neumann's idea:

Weather forecasting was, *par excellence*, a scientific problem suitable for solution using a large computer.

The objective of the project was to study the problem of predicting the weather by simulating the dynamics of the atmosphere using a digital electronic computer.

A Proposal for funding listed three "possibilities":

1. **Entirely new methods of weather prediction by calculation will have been made possible;**
2. **A new rational basis will have been secured for the planning of physical measurements and field observations;**
3. **The first step towards influencing the weather by rational human intervention will have been made.**

"Conference on Meteorology"

A "Conference on Meteorology" was arranged in the Institute for Advanced Studies (IAS), Princeton on 29–30 August, 1946.

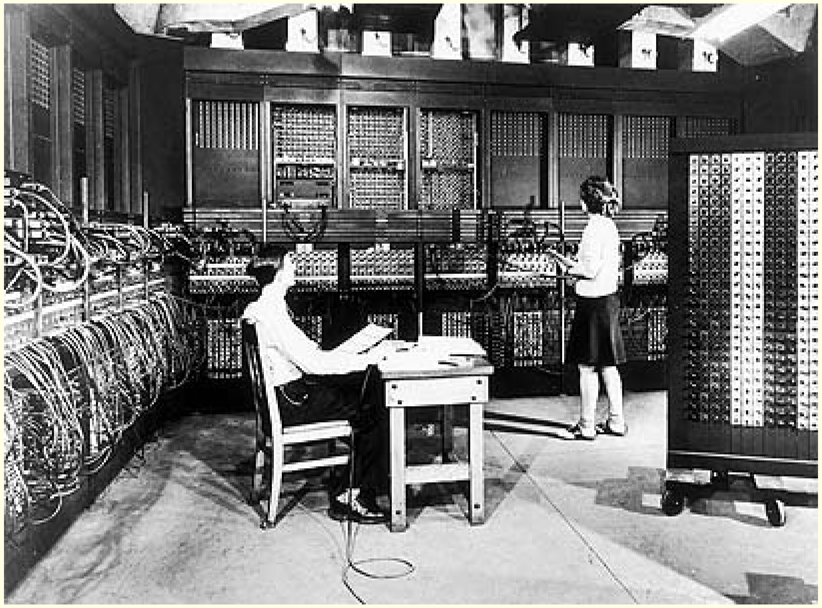
Participants included:

- **Carl Gustav Rossby**
- **Jule Charney**
- **George Platzman**
- **Norman Phillips**
- **Ragnar Fjørtoft**
- **Arnt Eliassen**
- **Joe Smagoinsky**
- **Phil Thompson**

Evolution of the Project:

- **Plan A: Integrate the Primitive Navier-Stokes Equations**
  Problems similar to Richardson's would arise
- **Plan B: Integrate baroclinic Q-G System**
  Too computationally demanding
- **Plan C: Solve barotropic vorticity equation**
  Very satisfactory initial results

The ENIAC



The **ENIAC** (Electronic Numerical Integrator and Computer) was the first multi-purpose programmable electronic digital computer.
It had:

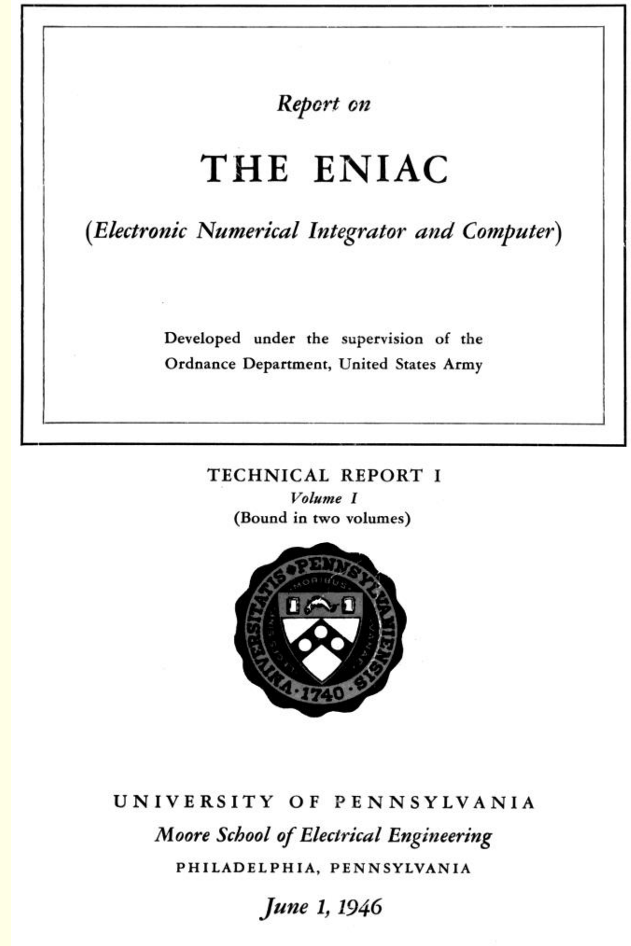
- 18,000 vacuum tubes
- 70,000 resistors
- 10,000 capacitors
- 6,000 switches

Power Consumption: 140 kWatts

The ENIAC: Technical Details.

ENIAC was a decimal machine. No high-level language. Assembly language. Fixed-point arithmetic: $-1 < x < +1$.

10 registers, that is,
Ten words of high-speed memory.
Function Tables:
624 6-digit words of "ROM", set on ten-pole rotary switches.
"Peripheral Memory":
Punch-cards.
Speed: FP multiply: 2ms (say, 500 Flops).
Access to Function Tables: 1ms.
Access to Punch-card equipment: You can imagine!



Report on

THE ENIAC

(*Electronic Numerical Integrator and Computer*)

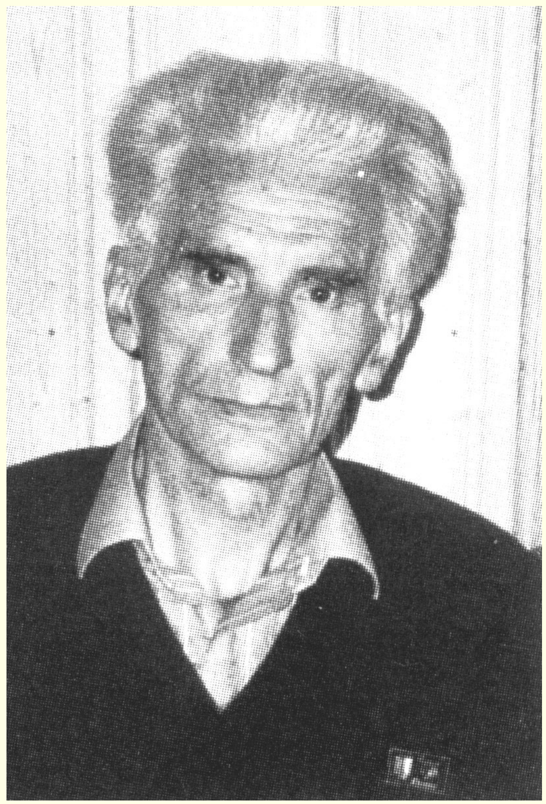
Developed under the supervision of the
Ordnance Department, United States Army

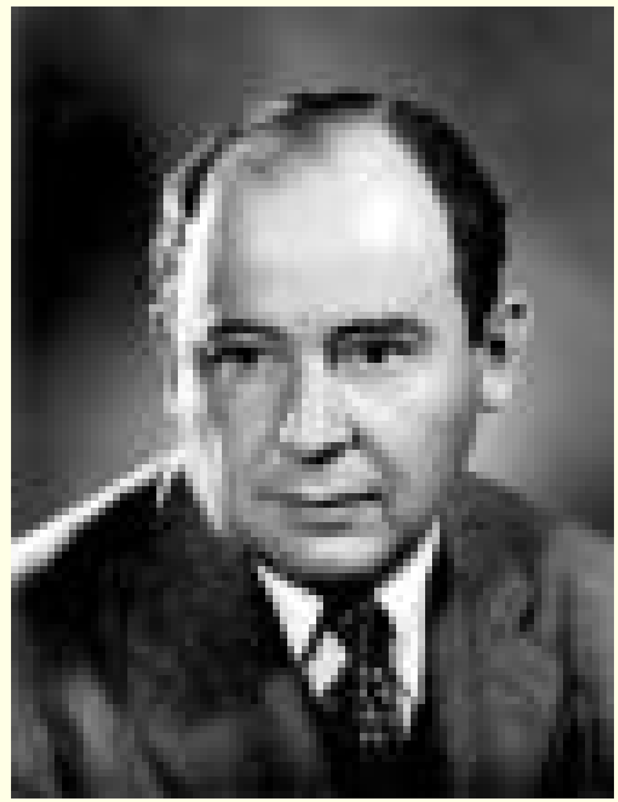
TECHNICAL REPORT I
Volume I
(Bound in two volumes)

UNIVERSITY OF PENNSYLVANIA
Moore School of Electrical Engineering
PHILADELPHIA, PENNSYLVANIA

June 1, 1946

Charney Fjørtoft von Neumann





Charney, et al., *Tellus*, 1950.

$$\begin{bmatrix}\textbf{Absolute}\\ \textbf{Vorticity}\end{bmatrix} = \begin{bmatrix}\textbf{Relative}\\ \textbf{Vorticity}\end{bmatrix} + \begin{bmatrix}\textbf{Planetary}\\ \textbf{Vorticity}\end{bmatrix} \qquad \eta = \zeta + f\,.$$

The atmosphere is treated as a single layer, and the flow is assumed to be nondivergent. Absolute vorticity is conserved following the flow.

$$\frac{d(\zeta + f)}{dt} = 0.$$

This equation looks deceptively simple. But it is nonlinear:

$$\frac{\partial \zeta}{\partial t} + \mathbf{V} \cdot \nabla(\zeta + f) = 0\,.$$

ENIAC Integrations, March, 1950

Five meteorologists started work in Aberdeen, MA, and continued day and night for 33 days.

- **Jule Charney**
- **Ragnar Fjørtoft**
- **John Freeman**
- **Joe Smagoinsky**
- **George Platzman**

One operation, calculation of the Jacobian, involved the reading of three punch-cards followed by a pause. As this sequence was repeated and repeated, Platzman wrote that the meteorologists could "dance a jig" to the rhythm.

Solution method for BPVE

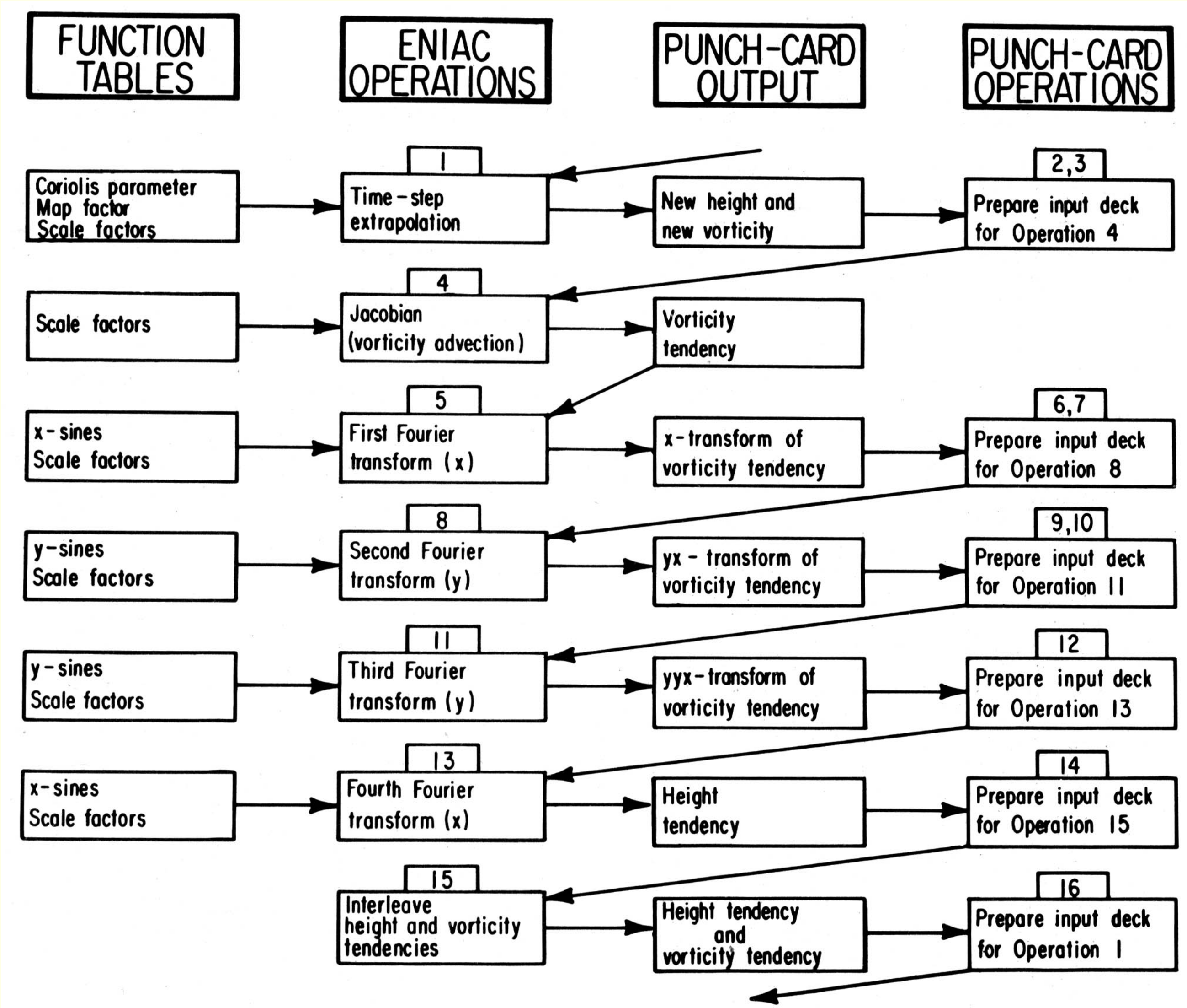
$$\frac{\partial \zeta}{\partial t} = \mathbf{J}(\psi, \zeta + f)$$

1. **Compute Jacobian**
2. **Step forward (Leapfrog scheme)**
3. **Solve Poisson equation for ψ (Fourier expansion)**
4. **Go to (1).**

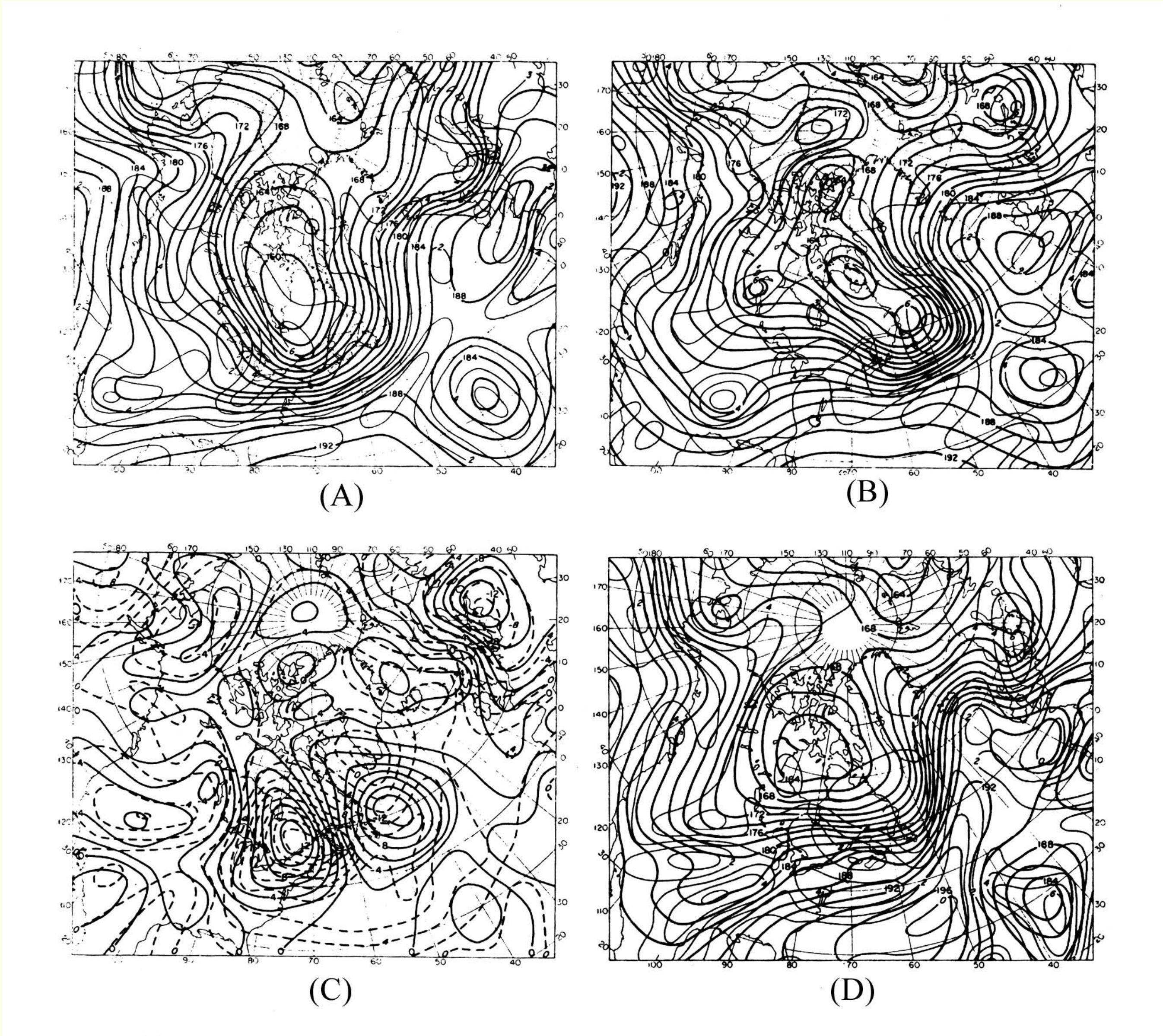
- **Timestep : Δt = 1 hour (2 and 3 hours also tried)**
- **Gridstep : Δx = 750 km (approximately)**
- **Gridsize : 18 x 15 = 270 points**
- **Elapsed time for 24 hour forecast: About 24 hours.**

Forecast involved punching about 25,000 cards. Most of the elapsed time was spent handling these.

ENIAC Algorithm



ENIAC: First Computer Forecast



Richardson's reaction

"Allow me to congratulate you and your collaborators on the remarkable progress which has been made in Princeton.

"This, although not a great success of a popular sort, is anyway an enormous scientific advance on the single, and quite wrong, result in which Richardson (1922) ended."

NWP Operations

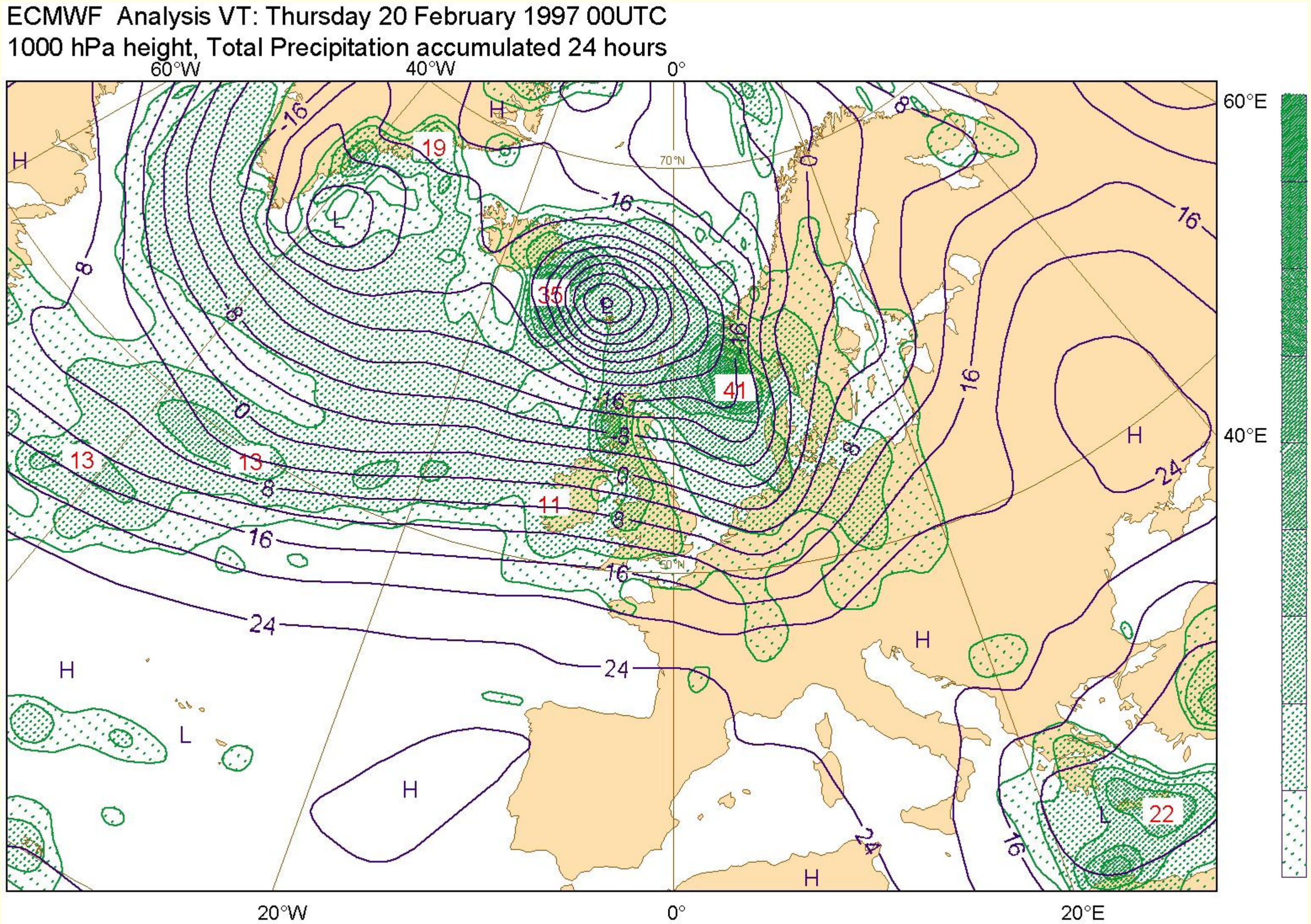
The Joint Numerical Weather Prediction (JNWP) Unit was established on July 1, 1954:

- **Air Weather Service of US Air Force**
- **The US Weather Bureau**
- **The Naval Weather Service.**

Operational numerical forecasting began on 15 May, 1955, using a three-level quasi-geostrophic model.

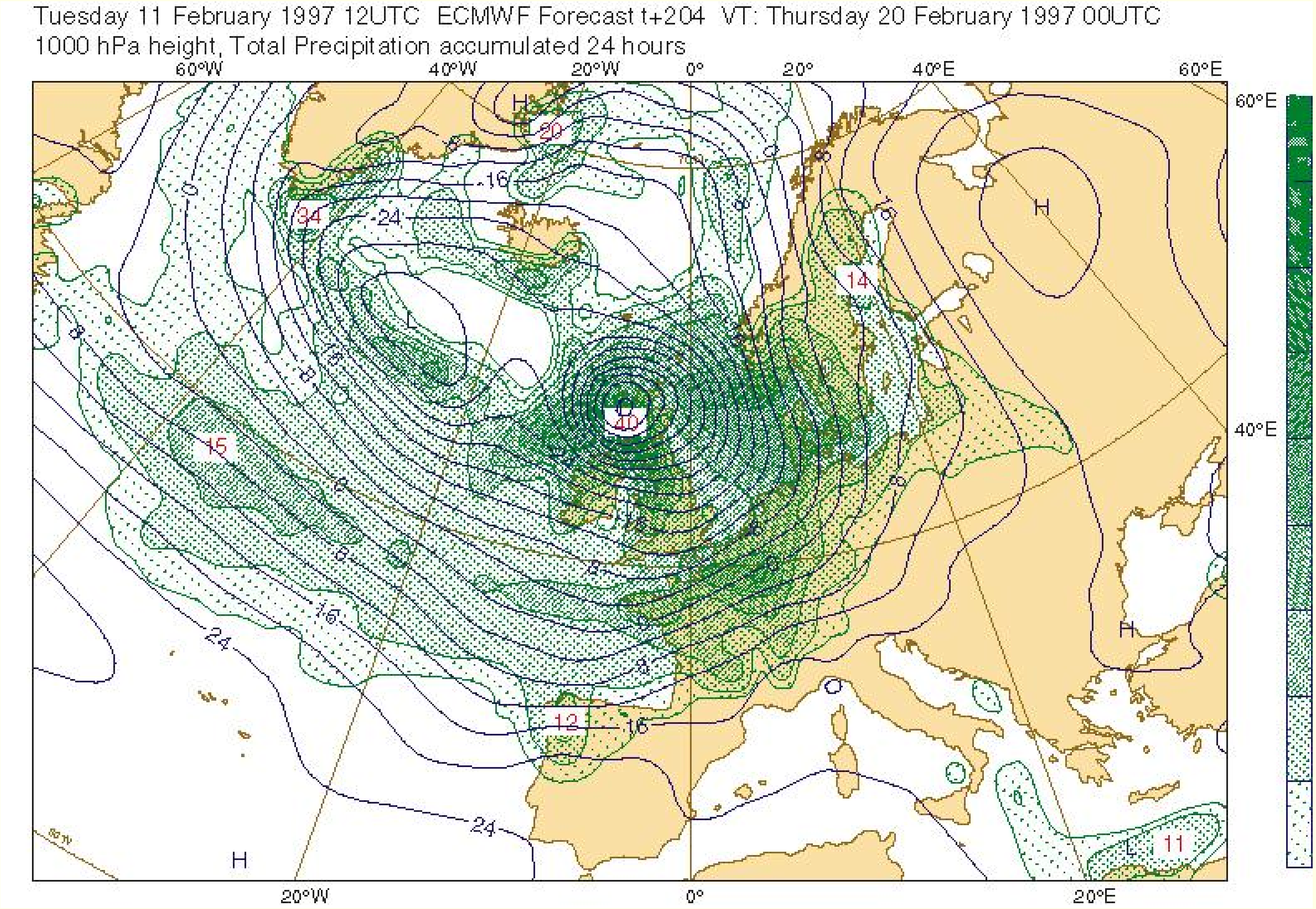
Computer Forecasting Today

Objective Analysis of Pressure



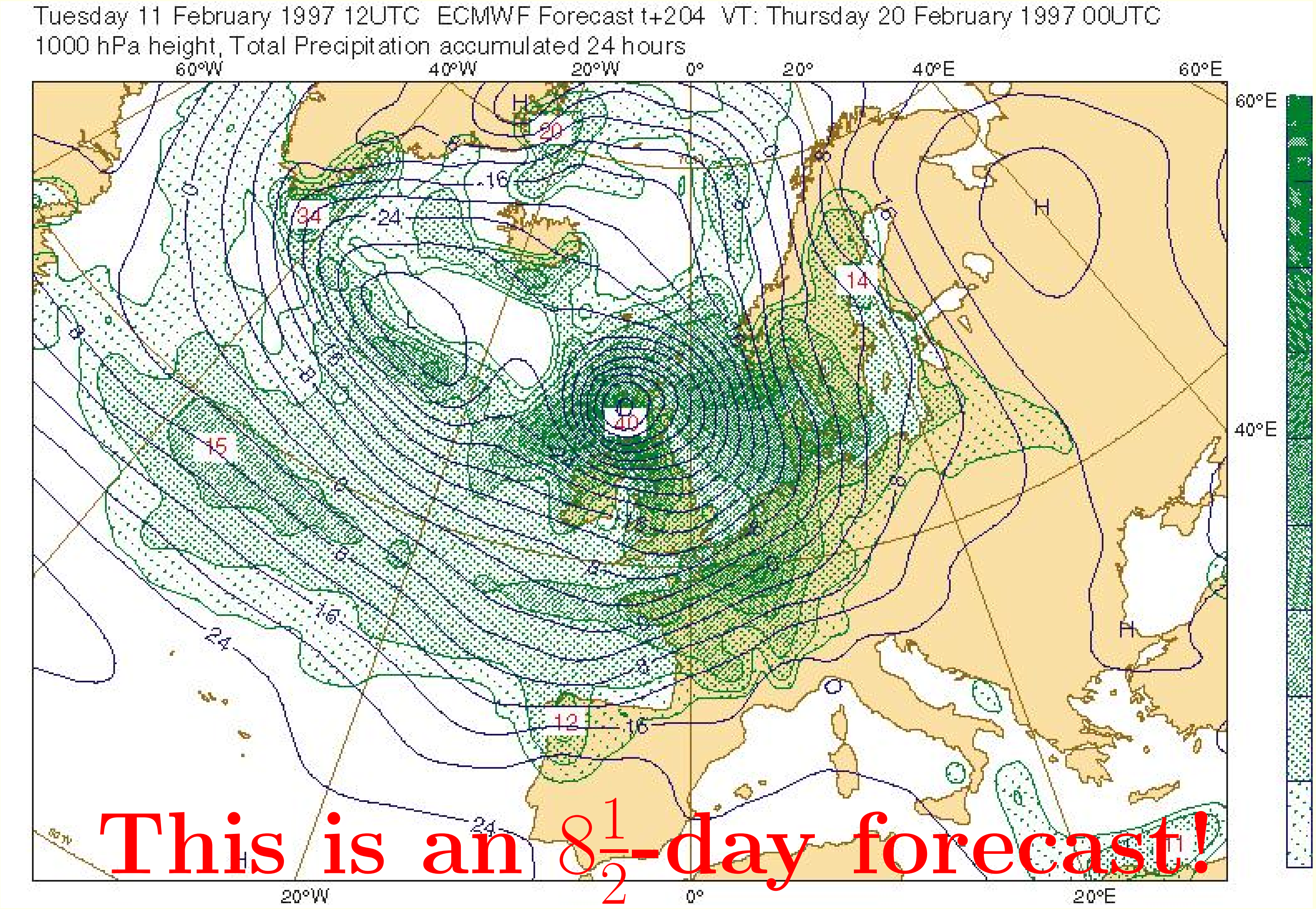
Analysis of 1000hPa height and 24hr precipitation.

Prediction of Surface Conditions



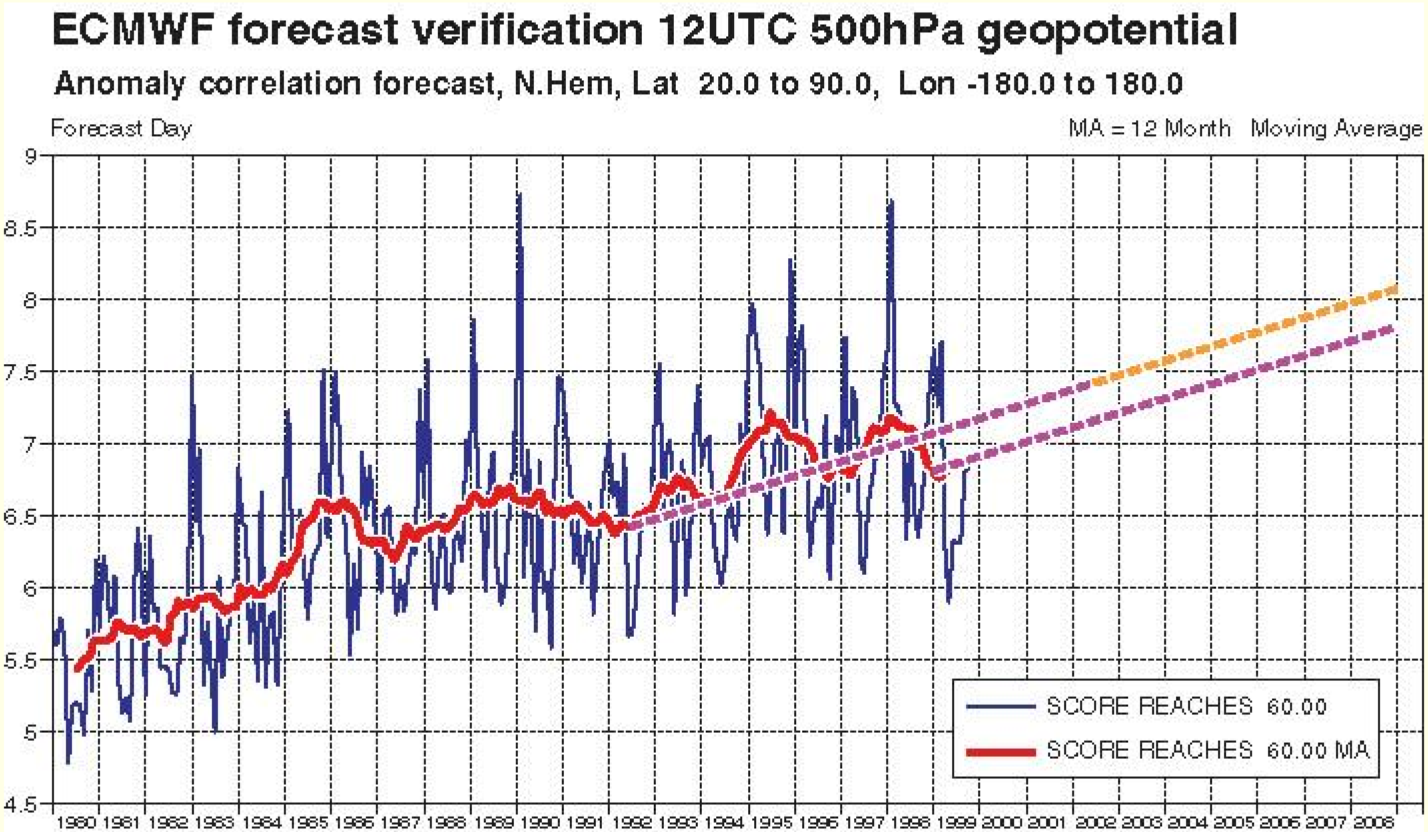
Forecast of 1000hPa height and 24hr precipitation.

Prediction of Surface Conditions



Forecast of 1000hPa height and 24hr precipitation.

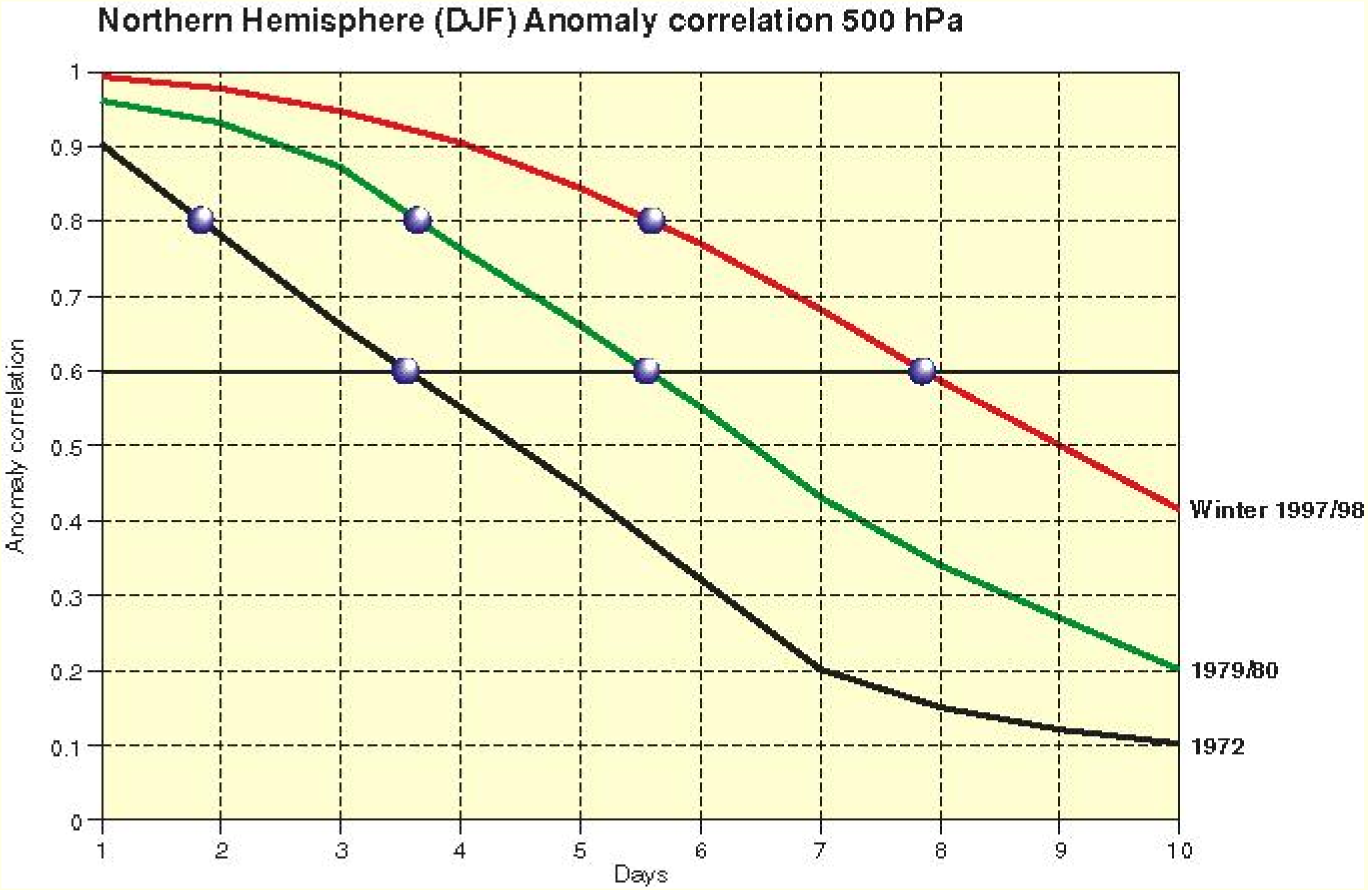
Objective Measure of Skill



Skill of 500 mb geopotential height. Forecast day when Anomaly Correlation falls to 0.6

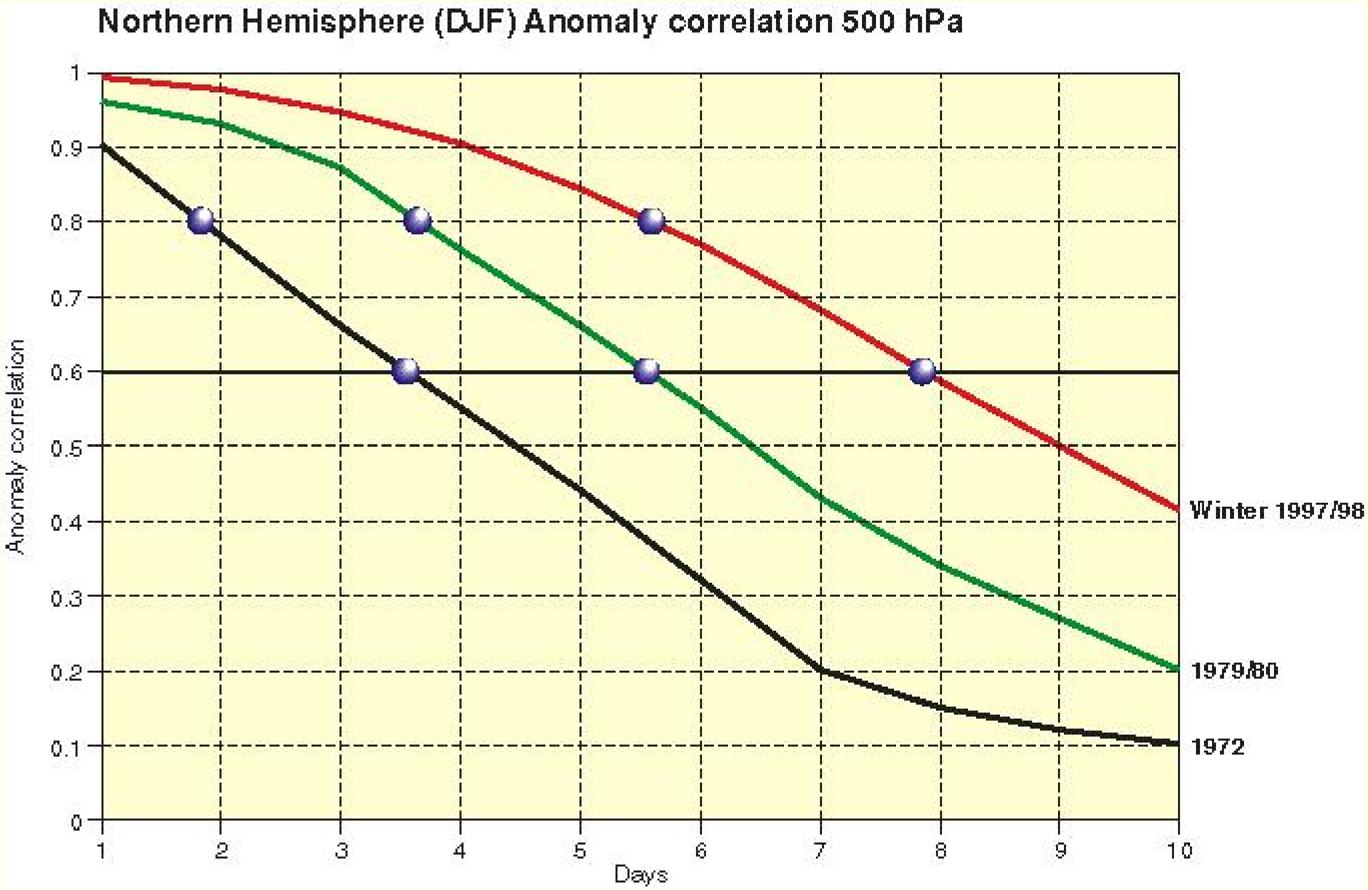
This is a measure of the useful forecast range.

Objective Measure of Skill



Comparative skill of 500 mb forecasts.

Objective Measure of Skill



Comparative skill of 500 mb forecasts.

The six-day forecasts now are as good as the two-day forecasts were in 1972.

Conclusions

- **Computer forecasts have improved dramatically since the ENIAC integrations of 1950.**

Conclusions

- **Computer forecasts have improved dramatically since the ENIAC integrations of 1950.**
- **NWP is an indispensible source of guidance for forecasters in preparing subjective forecasts**

Conclusions

- **Computer forecasts have improved dramatically since the ENIAC integrations of 1950.**
- **NWP is an indispensible source of guidance for forecasters in preparing subjective forecasts**
- **Prospects are excellent for further increases in accuracy and scope of NWP**

Conclusions

- Computer **forecasts have improved dramatically** since the ENIAC integrations of 1950.
- NWP is an **indispensible source of guidance** for forecasters in preparing subjective forecasts
- **Prospects are excellent** for further increases in accuracy and scope of NWP
- The mathematical equations developed by

  G G Stokes

  of Skreen are crucial in modelling and predicting atmospheric flow, and are thus

  the key to modern weather forecasting.

The End

Typesetting Software: TEX, *Textures*, LATEX, `hyperref`, `texpower`, Adobe Acrobat 4.05
Graphics Software: Adobe Illustrator 9.0.2
LATEX Slide Macro Packages: Wendy McKay, Ross Moore