Image Processing Emerges from the Shadows

Determining the Time and Date from Shadows in a Photograph

Peter Lynch
School of Mathematics & Statistics
University College Dublin

★ ★ ★

Applied Maths Meetup,
Riot Games, Dublin.

June 23, 2022

Outline

Abstract

A photograph of Kingstown (Dun Laoghaire) taken from the East Pier includes some children. Their shadows in the morning Sun enable us to determine the time of day when the photograph was taken and also permit us to estimate the season. The time of day (local solar time) determined by the azimuthal angle of the shadows can be estimated with confidence. The season is determined by the azimuth and elevation together, and is subject to greater uncertainty. Moreover, since any combination of these angles can occur twice each year, we must use other evidence to decide which of two dates is more likely.

The conclusion of the analysis is that the photograph was taken at approximately 08:20 Local Solar Time. The most probable times of year are mid-April or late August, approximately nine weeks before and nine weeks after the Summer solstice. The ample foliage on the trees visible in the photograph might be used as evidence in support of an August date.

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

The Problem

My neighbour, Brian Ellis, gave me the photo.

For genealogical and historical reasons,
he wished to know the time and date
when the photograph was taken.

The photo was taken on a bright sunny morning.

The key lies in the shadows cast by the children.

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

The Post Card

- Photograph taken c. 1905.
- Spire of the Mariners' Church
- Tower of the Royal Marine Hotel
- Spire of Saint Michael's Church
- Tower of the Town Hall (behind smoke)
- Bandstand on the East Pier
- Old RNLI Lifeboat House
- George IV Obelisk

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

The Main Clues

The ship at Carlisle Pier is building up steam.
The scheduled sailing time was 9:00 AM.

There are four children on the pier.
All the children cast clear shadows.

The lengths and angles are the key to deducing the time and date of the photo.

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

Location of the Camera

The camera position can be determined accurately from vertical alignments.

The photographer was standing on the upper deck.

There are several vertical alignments:

- The spire of Saint Michael's Church
- The spire of the Mariners' Church

Using these, it is easy to determine the location of the photographer.

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

Pencil of Lines on the Pier

The following three lines are parallel:

1. **The edge of the upper deck, visible in the lower left corner of the image.**
2. **The shadow of the upper deck, visible on the lower deck.**
3. **The outer edge of the lower deck.**

All these lines meet at a vanishing-point.
All the lines through this point form a pencil.

The direction of the pier was read from an OSI map: the compass bearing is 30° East of True North.

Measurement of the Angles

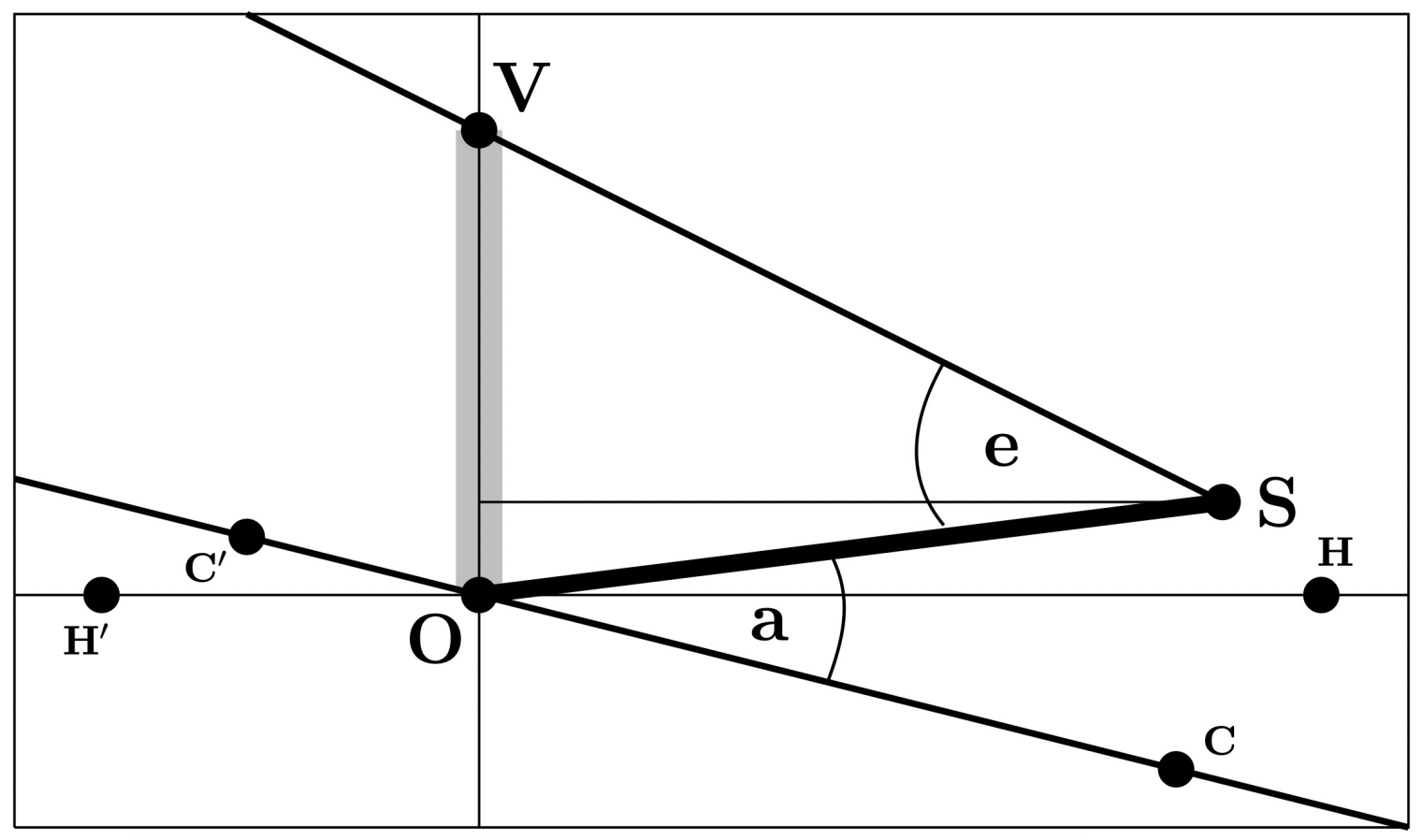

Figure: Schematic diagram of principal lines and angles.

Adjustment of the Angles

The boy in the centre of the photograph — Reggie — was chosen for closer examination, as his shadow was the one most clearly defined.

Angles must be adjusted for camera viewing angle. True representation of an angle in the horizontal plane requires the camera to be vertically above it.

The camera is at a low viewing angle: its height above the lower deck is about 3.5 metres.

The distance to Reggie is about 20 metres.

The viewing angle is $\kappa = 10°$.
We define $\gamma = (90° - \kappa) = 80°$.

Measurement of the Angles

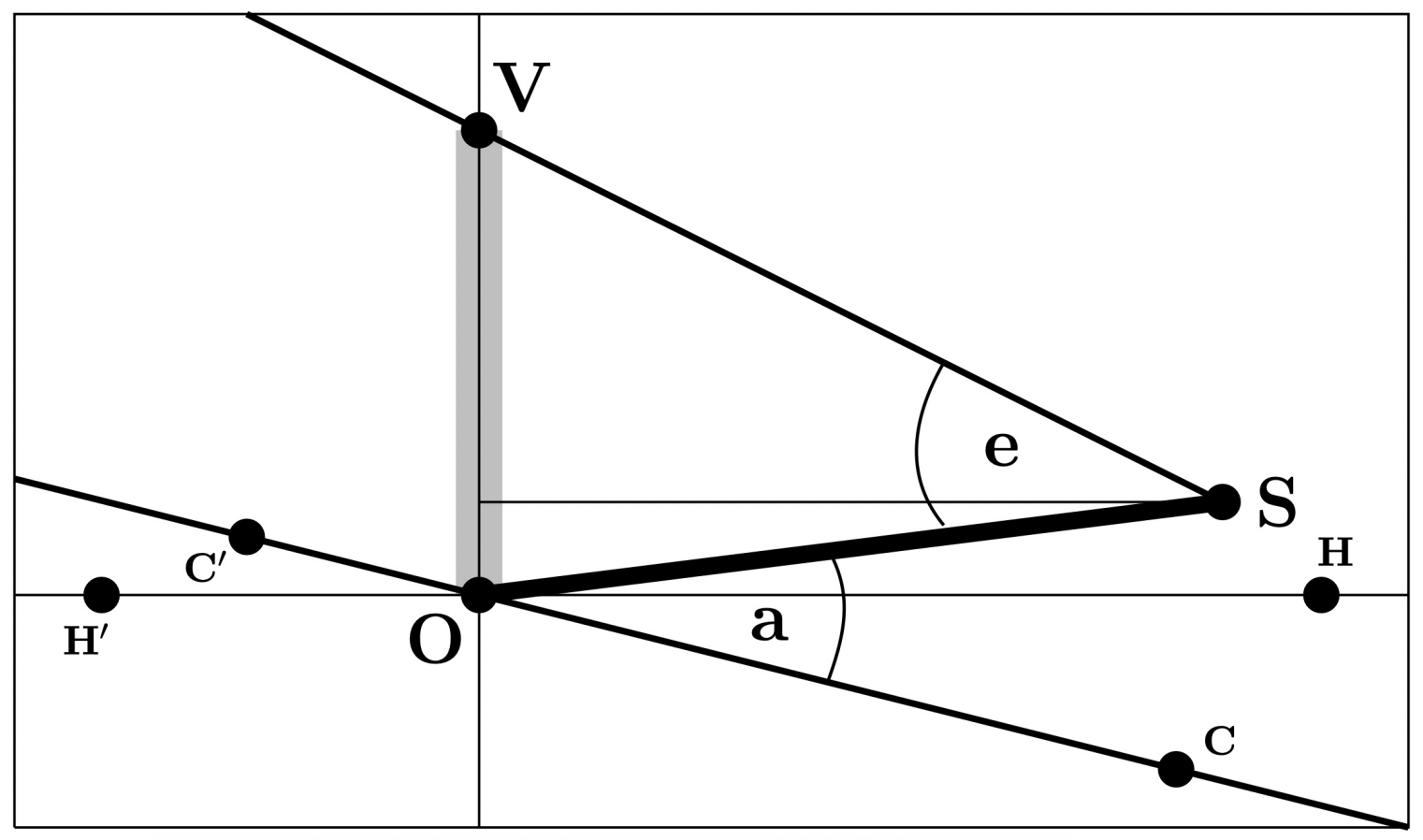

Figure: Schematic diagram of principal lines and angles.

Adjustment of the Angles

Assuming $\tan\psi$ is the tangent of an angle with one arm parallel to H′H, the foreshortening is

$$\tan\Psi = \frac{\tan\psi}{\cos\gamma}\,.$$

Then Ψ, greater in magnitude than ψ, is the true angle.

A similar adjustment was made for the elevation. The value obtained was $e = 31°$.

Assuming uncertainty of $\pm 10°$, the position of the Sun, given by the azimuth and elevation, is

$$(a, e) = (112 \pm 10°, 31 \pm 10°).$$

Determination of the Time

The local solar time can be determined once the azimuth and elevation of the Sun are known.

There are two formulas, one using the elevation e and one using the azimuth a.

$$\cos h = \frac{\sin e - \sin\varphi \sin\delta}{\cos\varphi \cos\delta} \qquad \sin h = \frac{\cos e \sin a}{\cos\delta}\,.$$

The angle δ is the *declination*. The two formulas were found to give very similar results.

With h in degrees, the local solar time (in hours) is

$$t = 12 - \frac{h}{15}$$

(15° corresponds to 1 hour).

Determination of the Time

Given the estimate $(a, e) = (112°, 31°)$ we deduce a declination of $\delta = 13°$.

Using the values $a = 112°$, $e = 31°$, $\varphi = 53.5°$ and $\delta = 13°$, both the formulæ give time 8:21 LST.

Solar time in Kingstown was close to Dunsink Time (Dublin Mean Time (DMT), the legal time for Ireland.

DMT was about 25 minutes behind Greenwich Mean Time (GMT), now usually denoted UTC.

Thus, the photograph was taken at about 08:45 UTC.

Possible Positions of the Sun

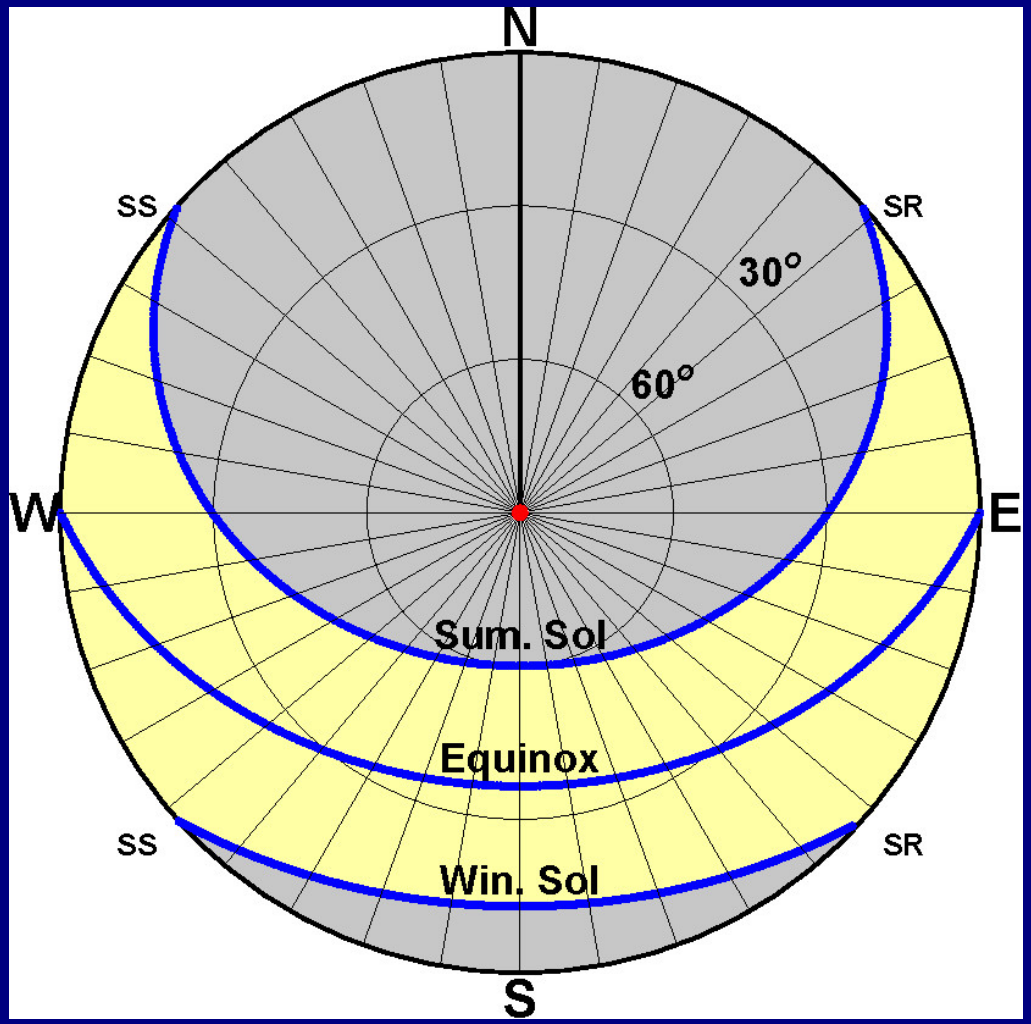

Figure: Path of the Sun on the solstices and equinoxes. The light region represents all possible positions of the Sun.

Possible Positions of the Sun

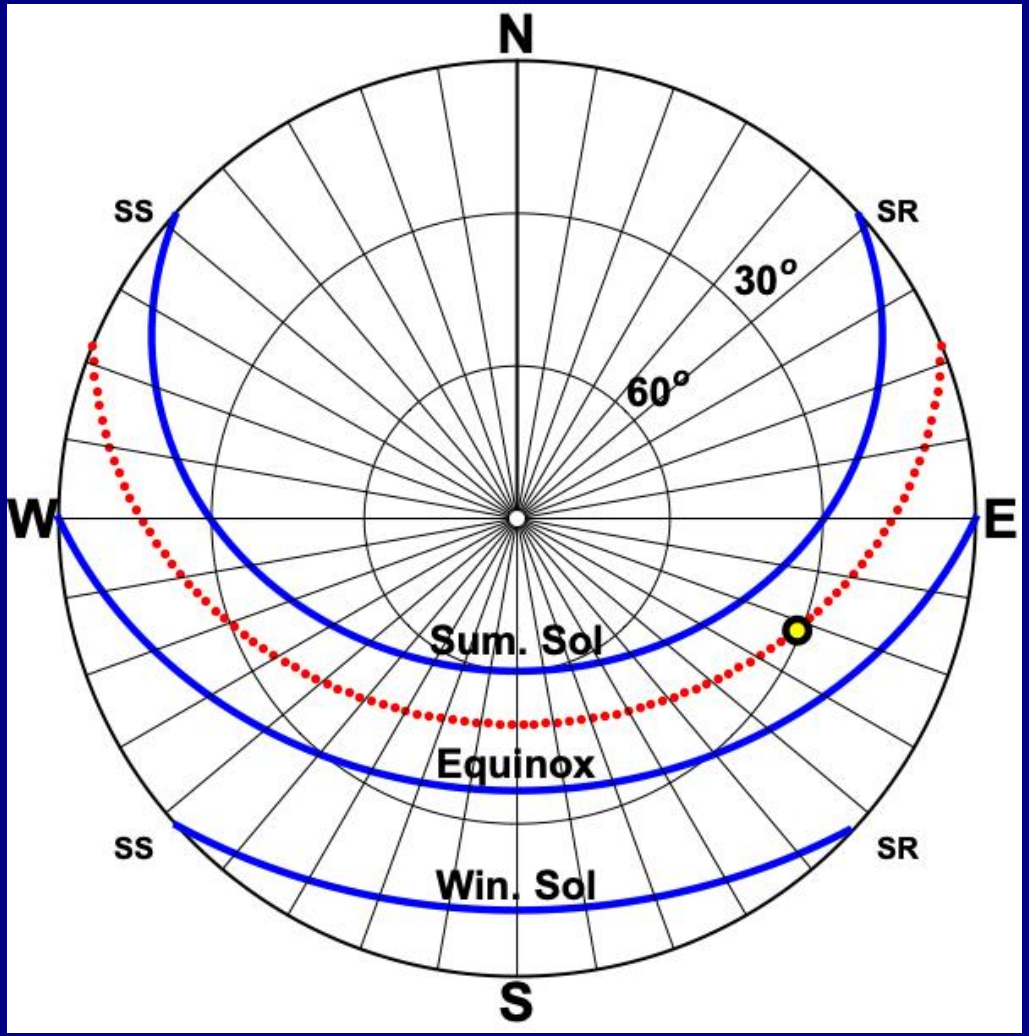

Figure: The dotted red line is the path for estimated value of the declination, $\delta = 13°$.

Possible Positions of the Sun

The position of the Sun in the sky depends on where we are and on the time of day.

Due to the Earth's rotation, the Sun appears to cross the celestial sphere each day along the *ecliptic*.

The path of the Sun depends on the latitude, while the time when the Sun crosses the local meridian is determined by the longitude.

Declination

Plotting the elevation e and azimuth a on a polar diagram, we obtain a plot of the Sun as it moves across the sky.

Over a year, the midday solar elevation at Dun Laoghaire varies from $60°$ (Summer) to $13°$ (Winter).

If we know the azimuth and elevation, (a, e), we can deduce the date, but this not unambiguously:

For every point within the accessible region, there are *two* times each year when the Sun is at that point.

Declination

The declination δ is the latitude at which the Sun appears vertically overhead at local Noon.

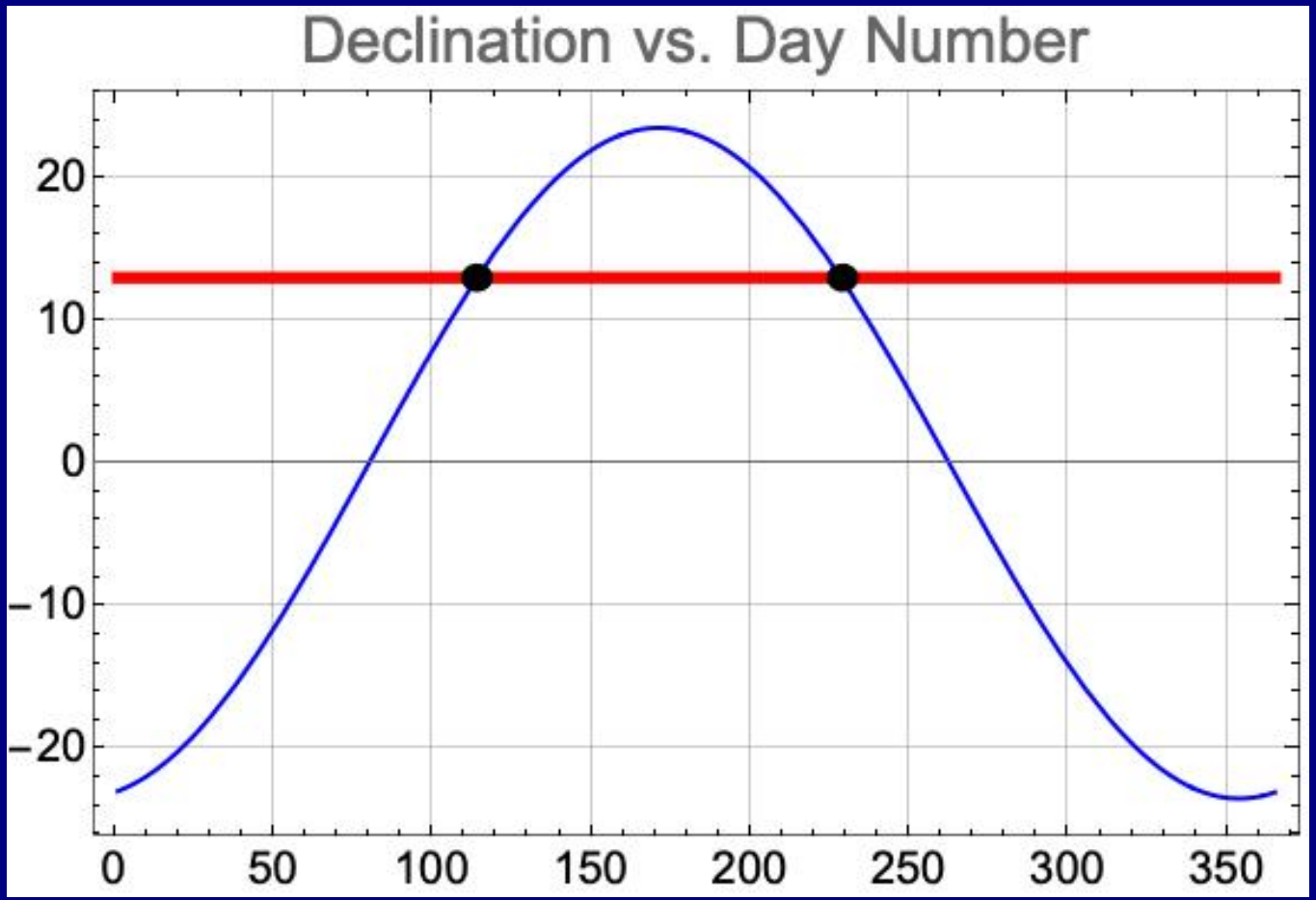

Figure: Declination of the Sun as a function of day number.

Declination

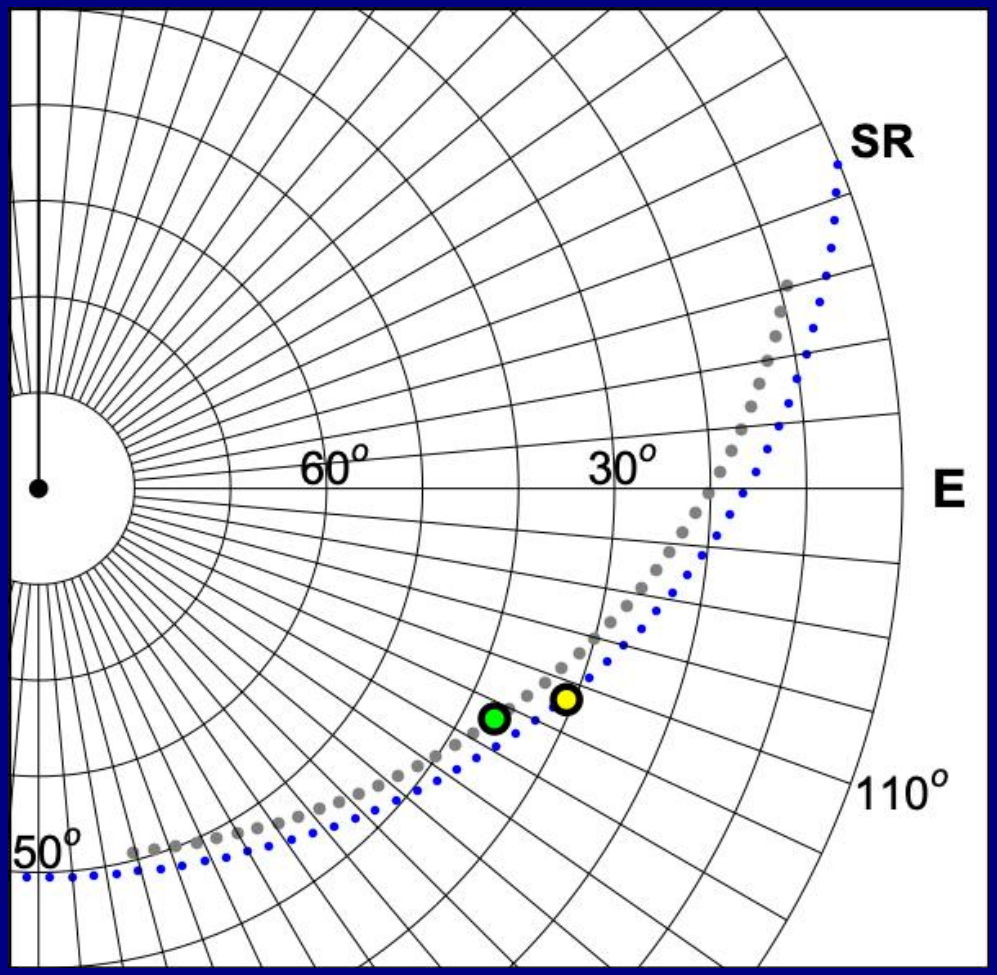

Figure: Path of the Sun for a declination $\delta = 13°$ and $(a, e) = (112°, 31°)$. Yellow circle: calculated position. Green circle: position for the reconstructed image.

Deducing the Date

The next step is to deduce the date from the declination:

$$\delta = 23.5^\circ \times \sin\left[\left(\frac{d - d_0}{365}\right) 360^\circ\right]$$

(d is the day number. $d_0 = 80$ is the Spring equinox).

We can invert this to deduce the day number:

$$d = d_0 + \left(\frac{365}{360}\right) \arcsin\left(\frac{\delta}{23.5}\right) .$$

There are two possible solutions; ('arcsin' has two values).

Deducing the Date

We find the value $d = 80 + 34 = 114$ which is April 24.

This is 58 days before the Summer solstice.

The second solution is August 18.

The uncertainty is estimated to be about 15 days.

Deducing the Date

We find the value $d = 80 + 34 = 114$ which is April 24.

This is 58 days before the Summer solstice.

The second solution is August 18.

The uncertainty is estimated to be about 15 days.

To support the analysis, a photograph was taken from the East Pier, close to the estimated date and time.

The Photograph

Figure: Dun Laoghaire taken at 09:48 IST, 21 April, 2022.

The Postcard

Figure: Postcard photograph of Kingstown. c. 1905.

Conclusion

The position of the Sun was estimated by analysis of shadows in the postcard image.

The estimated local solar time and date were about 08:20 LST and mid-April or late August.

Which of these dates is more likely?

The ample foliage visible in the photograph might point towards the August date.

References

- Lynch, Peter, 2018: Pedro Nunes and the Retrogression of the Sun. *Bull. Irish Math. Soc.*, 81, 23–30.

- NOAA Solar Calculator. NOAA Solar Calculator, Earth System Research Laboratories: https://gml.noaa.gov/grad/solcalc/.

- Wikipedia articles: (1) Position of the Sun. (2) Solar zenith angle. (3) Solar azimuth angle. (4) Declination.