# Partitioning the Wind in a Limited Domain

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ABSTRACT

The partitioning of a global windfield into rotational and divergent components is unique. These components are orthogonal and imply a corresponding partitioning of the kinetic energy. For a limited domain the partitioning is neither unique nor (necessarily) orthogonal and depends on the boundary conditions. Several simple boundary conditions are examined and the resulting wind components derived. A natural partitioning into three mutually orthogonal components, the rotational, divergent and harmonic components, is proposed. For a global domain the harmonic component vanishes, reducing the partitioning to the usual form.

## 1. Introduction

A horizontal windfield on the globe can be partitioned into rotational and divergent components in a unique way. These components are orthogonal—the areal integral of their product vanishes—and split the kinetic energy into corresponding parts. This analysis is a useful diagnostic technique which can provide valuable insight into the dynamics of the flow. It is of interest to investigate whether it can be applied when the wind information is available over only part of the globe.

In the case of wind data restricted to a limited area, the partitioning depends on the boundary conditions chosen for the streamfunction and velocity potential. A nonvanishing windfield which is both nondivergent and irrotational is possible, so the rotational and divergent components are not uniquely defined. Moreover, they are not necessarily orthogonal. In section 2 we derive the wind components for several simple boundary conditions and calculate the corresponding kinetic energy components. The dependence of the wind components on the boundary formulation is seen, and the nonorthogonality results in a nonpositive-definite cross-term in the expression for kinetic energy.

If wind components in a limited domain are to be useful for diagnostic analysis, they must be defined in an unambiguous way. A perspicuous partitioning into three parts, the rotational, divergent and harmonic components, is proposed in section 3. The rotational and divergent parts are specified by assuming that the corresponding stream function and velocity potential vanish on the boundary. They are orthogonal, and minimize the rotational and divergent components of kinetic energy. The residual wind, the harmonic component, is both irrotational and nondivergent and may be described by either a streamfunction or a velocity potential (which are harmonic conjugates). The harmonic component is orthogonal to the other two components and gives a three-way splitting of wind and kinetic energy. On a global domain the harmonic component vanishes and the partitioning reduces to the usual two-component form.

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## Two-component partitioning

### a. Theory

We assume a horizontal windfield $\mathbf{V}$ given on a domain $\Omega$ bounded by $\partial\Omega$. The vorticity $\zeta$ and divergence $\delta$ are defined by

$$\zeta = \mathbf{k}\cdot\nabla\times\mathbf{V}, \quad \delta = \nabla\cdot\mathbf{V}. \tag{1}$$

By means of the Helmholtz theorem (Morse and Feshbach 1953), the windfield may be expressed in terms of a stream function $\psi$ and velocity potential $\chi$

$$\mathbf{V} = \mathbf{V}_\chi + \mathbf{V}_\psi = \nabla\chi + \mathbf{k}\times\nabla\psi. \tag{2}$$

From (1) and (2) we immediately obtain Poisson equations for $\chi$ and $\psi$

$$\nabla^2\chi = \delta, \quad \nabla^2\psi = \zeta. \tag{3a,b}$$

Boundary conditions on $\partial\Omega$ are required to determine the solutions. From (2), the wind components normal and tangential to the boundary are

$$V_n \equiv \mathbf{n}\cdot\mathbf{V} = -\frac{\partial\psi}{\partial s} + \frac{\partial\chi}{\partial n} \tag{4a}$$

$$V_s \equiv \mathbf{s} \cdot \mathbf{V} = \frac{\partial \psi}{\partial n} + \frac{\partial \chi}{\partial s} \tag{4b}$$

where $\mathbf{s}$ and $\mathbf{n}$ are tangent and normal unit vectors and $s$ and $n$ are distances along and normal to the boundary (measured anticlockwise and outward, respectively).

If the definitions (1) are integrated over $\Omega$ and (4) used, the following compatibility conditions result from Gauss' and Stokes' theorems:

$$\iint_\Omega \delta da = \oint_{\partial\Omega} V_n ds = \oint_{\partial\Omega} \frac{\partial \chi}{\partial n} ds \tag{5a}$$

$$\iint_\Omega \zeta da = \oint_{\partial\Omega} V_s ds = \oint_{\partial\Omega} \frac{\partial \psi}{\partial n} ds. \tag{5b}$$

If Neumann boundary conditions are used for $\chi$ or $\psi$, they must be consistent with (5a) or (5b).

In Lynch (1988, Appendix) it was shown that either $\chi$ or $\psi$ could assume arbitrary values on $\partial\Omega$ and that consequently the irrotational and nondivergent components of $\mathbf{V}$ are not uniquely defined. Sangster (1960) proposed the following method of solution: [1] set $\chi_B = 0$ (subscript $B$ denotes the value on $\partial\Omega$) and solve (3a) for $\chi$; [2] integrate (4a) around $\partial\Omega$ to get $\psi_B$; [3] solve (3b) for $\psi$. Only the normal component of the boundary wind is used.

TABLE 1. Eight variations of Sangster's method for computing $\chi$ and $\psi$. The first boundary condition assumes a constant value or gradient. The second is derived from (4a) or (4b). The first four versions use the normal boundary wind, the last four use the tangential component. Dirichlet and Neumann conditions are denoted by D and N respectively. $\gamma_n$ and $\gamma_s$ are defined in (6a) and (6b).

| Version number | First boundary condition | Second boundary condition | Type of first and second B.C. |
|---|---|---|---|
| 1 | $\chi = 0$ | $\psi = \int_{s_0}^{s} \left(\frac{\partial \chi}{\partial n} - V_n\right) ds$ | D D |
| 2 | $\frac{\partial \chi}{\partial n} = \gamma_n$ | $\psi = \int_{s_0}^{s} (\gamma_n - V_n) ds$ | N D |
| 3 | $\psi = 0$ | $\frac{\partial \chi}{\partial n} = V_n$ | D N |
| 4 | $\frac{\partial \psi}{\partial n} = \gamma_s$ | $\frac{\partial \chi}{\partial n} = \left(V_n + \frac{\partial \psi}{\partial s}\right)$ | N N |
| 5 | $\chi = 0$ | $\frac{\partial \psi}{\partial n} = V_s$ | D N |
| 6 | $\frac{\partial \chi}{\partial n} = \gamma_n$ | $\frac{\partial \psi}{\partial n} = \left(V_s - \frac{\partial \chi}{\partial s}\right)$ | N N |
| 7 | $\psi = 0$ | $\chi = \int_{s_0}^{s} \left(V_s - \frac{\partial \psi}{\partial n}\right) ds$ | D D |
| 8 | $\frac{\partial \psi}{\partial n} = \gamma_s$ | $\chi = \int_{s_0}^{s} (V_s - \gamma_s) ds$ | N D |

We will consider eight variations of Sangster's method to derive the $\chi$ and $\psi$ fields. The details of these variations are collected in Table 1. The first four versions use $V_n$ on $\partial\Omega$, the last four use $V_s$. For each of these groups a simple boundary condition (the *first* B.C.) is assumed for $\chi$ or $\psi$ and this may be a Dirichlet or Neumann condition. Either (3a) or (3b) is solved using this condition on $\partial\Omega$. The Dirichlet conditions may be homogeneous, but the Neumann conditions must satisfy (5a) or (5b). The simplest way to ensure this is to assume constant gradient conditions

$$\frac{\partial \chi}{\partial n} = \gamma_n \equiv \frac{1}{L} \oint_{\partial\Omega} V_n ds \quad \text{on } \partial\Omega \tag{6a}$$

or

$$\frac{\partial \psi}{\partial n} = \gamma_s \equiv \frac{1}{L} \oint_{\partial\Omega} V_s ds \quad \text{on } \partial\Omega \tag{6b}$$

where $L$ is the length of the boundary.

Once the first boundary condition is fixed and a Poisson equation solved, the *second* condition is derived from either (4a) (for versions 1 to 4) or (4b) (for versions 5 to 8). For versions 3 to 6 Neumann conditions follow immediately from (4a) or (4b). For the remaining versions (4a) or (4b) is integrated from an arbitrary point $s_0$ on $\partial\Omega$, to give $\psi(s)$ or $\chi(s)$ on the boundary. The remaining Poisson equation may then be solved.

We define the kinetic energy (assuming density $\rho = 1$) by

$$K \equiv \iint_\Omega \tfrac{1}{2} \mathbf{V} \cdot \mathbf{V} da$$

and its divergent and rotational components by

$$K_\chi \equiv \iint_\Omega \tfrac{1}{2} \mathbf{V}_\chi \cdot \mathbf{V}_\chi da; \quad K_\psi \equiv \iint_\Omega \tfrac{1}{2} \mathbf{V}_\psi \cdot \mathbf{V}_\psi da.$$

The kinetic energy may be expanded, using (2):

$$K = K_\chi + K_\psi + \iint_\Omega \mathbf{V}_\chi \cdot \mathbf{V}_\psi da.$$

If the domain $\Omega$ is limited, the cross-product term will generally not vanish. Using (2) again and Stokes' theorem we may write it as

$$K_{\chi\psi} \equiv \iint_\Omega \mathbf{V}_\chi \cdot \mathbf{V}_\psi da = \oint_{\partial\Omega} \psi \frac{\partial \chi}{\partial s} ds = -\oint_{\partial\Omega} \chi \frac{\partial \psi}{\partial s} ds. \tag{7}$$

This term may be positive or negative, and the kinetic energy does not partition into divergent and rotational components. For the odd-numbered versions in Table

1 either $\chi_B = 0$ or $\psi_B = 0$, the cross-term vanishes and the partitioning of $K$ is well defined.

A condition for minimizing the energy components is easily derived (Pedersen 1971). If we consider a given $\chi$ field, it may be split into two parts $\chi = \chi_1 + \chi_2$ such that

$$\nabla^2\chi_1 = \delta, \quad \chi_1 = 0 \quad \text{on } \partial\Omega$$

$$\nabla^2\chi_2 = 0, \quad \chi_2 = \chi_B \quad \text{on } \partial\Omega$$

where $\chi_B$ is the value assumed by $\chi$ on $\partial\Omega$. The divergent kinetic energy also splits into two parts, $K_\chi = K_1 + K_2$, where

$$K_1 = \iint_\Omega \frac{1}{2}\nabla\chi_1\cdot\nabla\chi_1 da; \quad K_2 = \iint_\Omega \frac{1}{2}\nabla\chi_2\cdot\nabla\chi_2 da$$

(by means of the divergence theorem the term involving $\nabla\chi_1\cdot\nabla\chi_2$ is easily shown to vanish). Since both parts are positive-definite and $K_1$ is independent of the boundary values, $K_\chi$ will be a minimum for $\chi_B$ constant, in which case $K_2 = 0$. In a similar way a constant boundary condition for $\psi$ minimizes $K_\psi$. In either of these cases, which include the odd-numbered versions in Table 1, the partitioning of kinetic energy is meaningful, since the cross-term $K_{\chi\psi}$ vanishes.

Conditions for minimizing energy components may also be derived using a variational formulation. For example, using (2) we may write the divergent kinetic energy as the difference, in a least-squares sense, between the total wind and the nondivergent component:

$$K_\chi = \iint_\Omega \frac{1}{2}(\mathbf{V} - \mathbf{V}_\psi)^2 da.$$

If $\psi$ is chosen to minimize this functional, the resulting Euler-Lagrange equation is (3b) and the natural boundary condition is the Neumann condition

$$\frac{\partial\psi}{\partial n} = V_s$$

(Lanczos 1966). It follows from (4b) that $\chi$ must be constant on the boundary; taking this arbitrary constant to be zero, we get solution version 5. An analogous argument for minimizing $K_\psi$ leads to version 3. This variational approach has been used by Davies-Jones (1988) to obtain a stream function which is most like a pressure field at a fixed level.

### b. Numerical example

To illustrate the dependence of the wind components on the boundary conditions for $\chi$ and $\psi$, we calculate these components for the eight simple boundary formulations displayed in Table 1. The wind field is the (uninitialized) 500 hPa analysis for 0000 UTC, 22 November 1982. The region $\Omega$, covering Western Europe, the North Atlantic and Eastern Canada, can be seen in Lynch (1988; Fig. 1). Data are given on a rotated

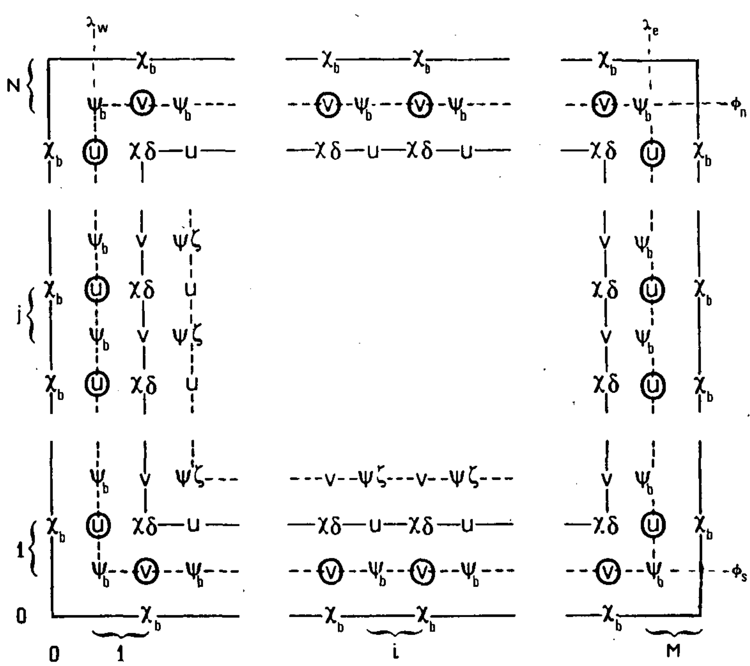


FIG. 1. Discrete grid used for versions 1 to 4. The normal boundary winds are encircled and outermost values of $\chi$ and $\psi$ are denoted by subscript $b$. For versions 5 to 8 the relative positions of the $\psi$ and $\chi$ boundaries are reversed and tangential winds on the $\chi$-boundary are used.

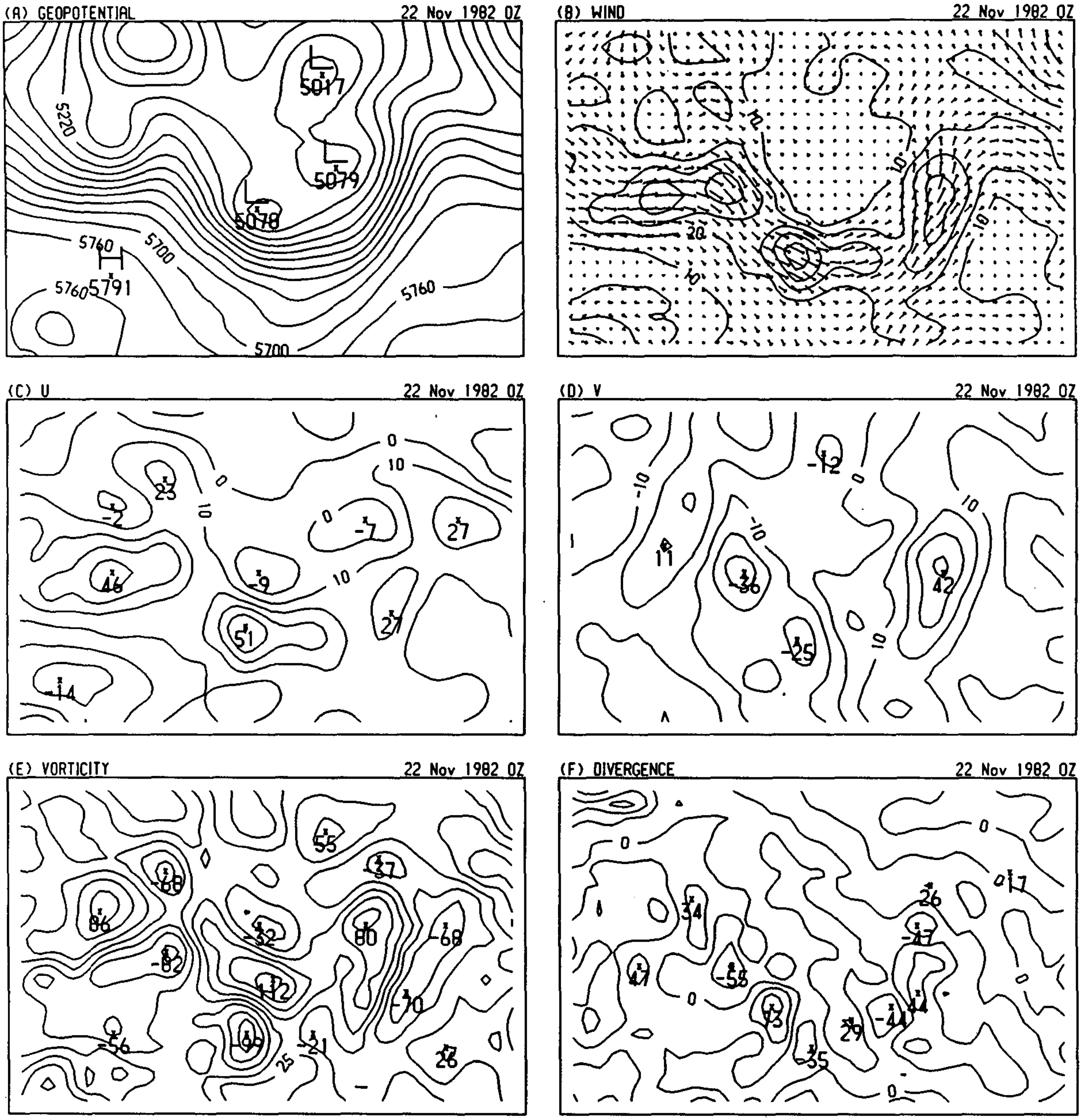


FIG. 2. Analysis for 0000 UTC, 22 November 1982. (A) 500 hPa geopotential (gpm). (B) wind direction and speed ($\mathrm{m\ s^{-1}}$). (C), (D) eastward and northward wind components. (E), (F) vorticity and divergence ($10^{-6}\ \mathrm{s^{-1}}$).

latitude/longitude grid (whose north pole is at the geographical position 30°N, 150°E), bounded by $(\lambda_W, \lambda_E) = (-40°, +38°)$ and $(\phi_S, \phi_N) = (-25°, +25°)$. The grid spacing is $\Delta\lambda = \Delta\phi = 2.0°$ and $40 \times 26$ points cover the area.

The discrete grid used to solve the Poisson equations for versions 1 to 4 is shown in Fig. 1. The $\chi$-boundary is outside the $\psi$-boundary, and normal winds on the latter are used. For versions 5 to 8 the relative positions of the $\psi$ and $\chi$ boundaries are reversed and tangential winds on the $\chi$-boundary are used.

The 500 hPa geopotential and wind are shown in Figs. 2A and B (for simplicity and clarity the fields are plotted on a rectangle without background maps). The dominant feature is a meandering westerly jetstream crossing the region. Wind components $u$ and $v$ are shown in Figs. 2C and 2D, and the corresponding vorticity and divergence in Figs. 2E and 2F. The vorticity

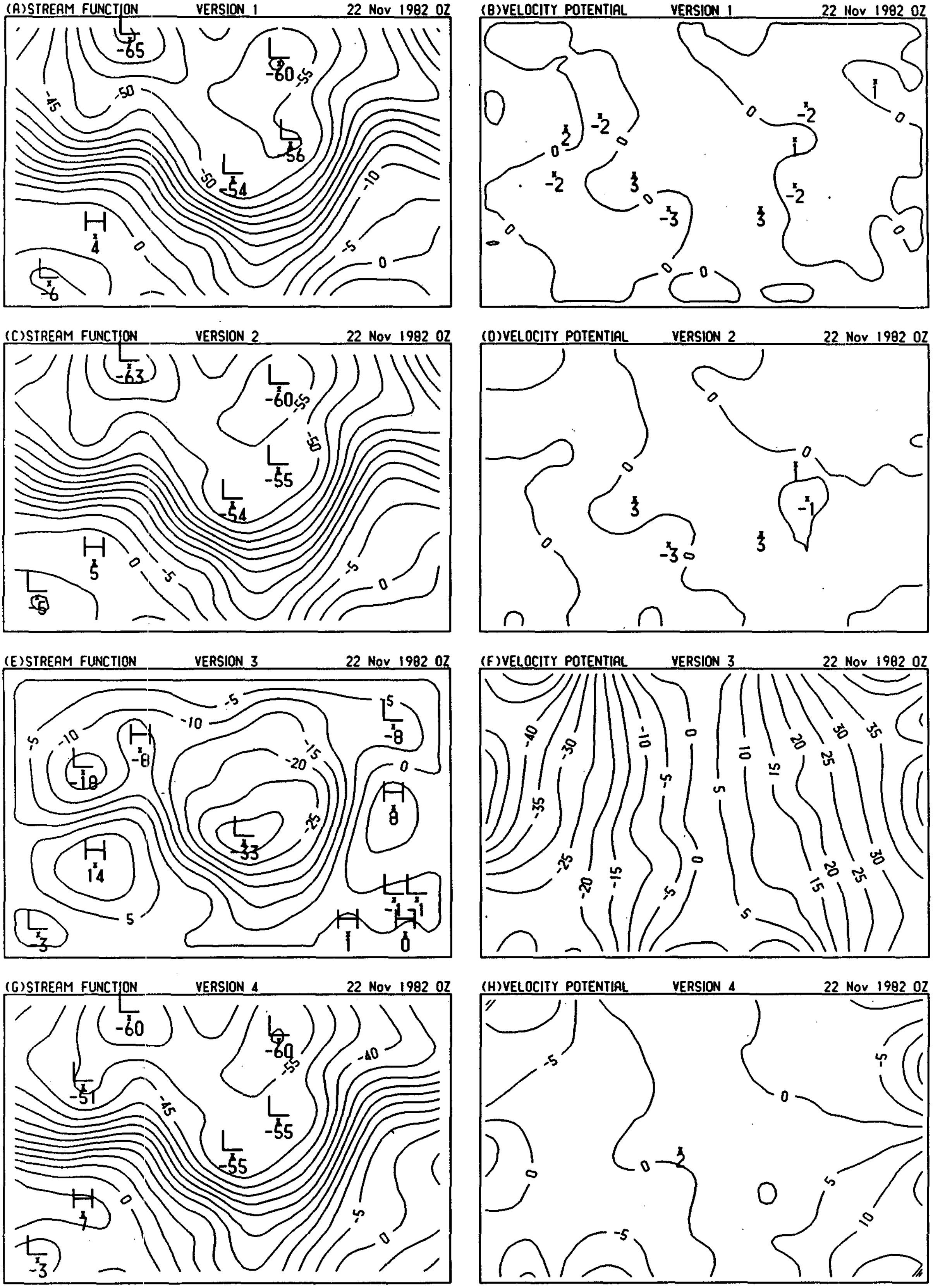


FIG. 3. Streamfunction (left panels) and velocity potential (right panels) derived using versions 1 to 4 of Table 1. The streamfunction is set to zero at the southwest corner. (Units $10^6$ m$^{-2}$ s$^{-2}$).

dominates, with a mean absolute value of $2.7 \times 10^{-5}$ $s^{-1}$, but the divergence is significant with mean absolute value of $9.9 \times 10^{-6}$ $s^{-1}$ and maxima concentrated along the jetstream.

In Fig. 3 we show the stream function (left panels) and velocity potential (right panels) for the first four versions (which use $V_n$ on $\partial\Omega$). The $\psi$ fields are similar for versions 1 and 2 (Figs. 3A and 3C) which assume $\chi_B = 0$ and $(\partial\chi/\partial n)_B = \gamma_n$, respectively ($\gamma_n = 0.27$ for the given data). The $\chi$ field is small in both cases. When $\psi_B = 0$ (version 3) the $\psi$ field is radically altered, as is the $\chi$ field (Figs. 3E and 3F). There is no cross-boundary rotational flow in this case, so the westerly jet must project mainly onto the divergent component. In version 4 (Figs. 3G and 3H) $\psi$ is similar to versions 1 and 2, except near the boundaries where it is forced to take the specified normal gradient $(\partial\psi/\partial n)_B = \gamma_s$. For the given data $\gamma_s = 6.46$, forcing changes in the northwesterly flow at the eastern boundary, and elsewhere. These changes are also reflected in the $\chi$ field near the boundaries.

The fields resulting from versions 5 to 8 were also calculated. The $\chi$ boundary was moved in one grid step and the tangential wind component was used there. These fields are not shown, since they do not differ markedly from those already presented: version 5 gives results very similar to version 1, version 6 is similar to version 2, and so on.

The energy components were calculated for all eight versions and are given in Table 2. The values were normalized by the area and are in units $m^2\ s^{-2}$. The total kinetic energy differs for the first and last four versions, since the areas differ slightly. The rotational component $K_\psi$ dominates and is fairly constant except for versions 3 and 7, where $\psi_B = 0$ reduces it considerably. The divergent component is quite sensitive to the boundary conditions. Most noteworthy is the cross-term $K_{\chi\psi}$ which is nonzero for the even-numbered versions (Neumann *first* B.C.) and assumes both negative and positive values. Obviously, it has no physical meaning as a kinetic energy component, and this may be seen as a drawback to using these versions.

## 3. Three-component partitioning

### *a. Theory*

We have seen that the partitioning into irrotational and nondivergent components is not unique for a limited domain. In this section we show how to isolate the harmonic component and simultaneously minimize the divergent and rotational components of kinetic energy.

We define a *minimal* streamfunction $\hat{\psi}$ and velocity potential $\hat{\chi}$, both vanishing on $\partial\Omega$, by

$$\nabla^2\hat{\psi} = \zeta, \quad \hat{\psi}_B = 0 \tag{8a}$$

$$\nabla^2\hat{\chi} = \delta, \quad \hat{\chi}_B = 0. \tag{8b}$$

TABLE 2. Components of the kinetic energy resulting from a two way splitting of the wind field by solution versions 1 to 8 (units $m^2\ s^{-2}$, values normalized by total area).

| Version | $K_\chi$ | $K_\psi$ | $K_{\chi\psi}$ | $K_{\text{total}}$ |
|---|---|---|---|---|
| 1 | 4.41 | 185.03 | 0.0 | 189.44 |
| 2 | 6.27 | 182.32 | 0.85 | 189.44 |
| 3 | 79.17 | 110.27 | 0.0 | 189.44 |
| 4 | 20.58 | 185.82 | −16.96 | 189.44 |
| 5 | 4.10 | 188.12 | 0.0 | 192.22 |
| 6 | 4.71 | 194.85 | −7.33 | 192.22 |
| 7 | 74.17 | 118.05 | 0.0 | 192.22 |
| 8 | 17.88 | 187.55 | −13.21 | 192.22 |

The resulting rotational and divergent kinetic energy components take minimal values. With this convention we can refer to the wind components

$$\hat{\mathbf{V}}_\psi = \mathbf{k} \times \nabla\hat{\psi}, \quad \hat{\mathbf{V}}_\chi = \nabla\hat{\chi} \tag{9}$$

as *the* rotational and divergent components, avoiding the circumlocutions "nondivergent" and "irrotational." The harmonic component $\mathbf{V}_0$ which is both curl-free and divergence-free is defined by the residual

$$\mathbf{V}_0 = \mathbf{V} - \hat{\mathbf{V}} = \mathbf{V} - (\hat{\mathbf{V}}_\chi + \hat{\mathbf{V}}_\psi).$$

Since $\mathbf{V}_0$ is both irrotational and solenoidal, it can be expressed by *either* a stream function *or* a velocity potential

$$\mathbf{V}_0 = \mathbf{k} \times \nabla\psi_0 = \nabla\chi_0. \tag{10}$$

Both $\psi_0$ and $\chi_0$ satisfy the Laplace equation. They are harmonic conjugates, real and imaginary parts of the *complex* potential $\omega = \chi_0 - i\psi_0$, which satisfy the Cauchy-Riemann equations (Carrier et al. 1966)

$$\nabla\chi_0 - \mathbf{k} \times \nabla\psi_0 = \mathbf{0}$$

and form two orthogonal systems of isopleths:

$$\nabla\chi_0 \cdot \nabla\psi_0 = \mathbf{0}$$

[these properties follow trivially by using (10) in the expressions $(\mathbf{V}_0 - \mathbf{V}_0)$ and $(\mathbf{V}_0 \times \mathbf{V}_0)$]. They are calculated by specifying the normal or tangential component of $\mathbf{V}_0$ on $\partial\Omega$. If the normal component is used, solution version 1 of Table 1 gives $\psi_0$ and version 3 gives $\chi_0$. If the tangential wind is used, version 5 gives $\psi_0$ and version 7 gives $\chi_0$.

Let the windfield be partitioned into three components

$$\mathbf{V} = \mathbf{V}_0 + \hat{\mathbf{V}}_\chi + \hat{\mathbf{V}}_\psi \tag{11}$$

as defined above. The (minimal) divergent and rotational kinetic energy are given by

$$\hat{K}_\chi \equiv \iint_\Omega \frac{1}{2}\hat{\mathbf{V}}_\chi \cdot \hat{\mathbf{V}}_\chi da; \quad \hat{K}_\psi \equiv \iint_\Omega \frac{1}{2}\hat{\mathbf{V}}_\psi \cdot \hat{\mathbf{V}}_\psi da \tag{12}$$

and the harmonic component is defined by

$$K_0 \equiv \iint_\Omega \frac{1}{2} \mathbf{V}_0 \cdot \mathbf{V}_0 da. \qquad (13)$$

It is straightforward to show that the components of (11) are pairwise orthogonal, so that no cross-terms occur and the kinetic energy splits into three parts

$$K = K_0 + \hat{K}_\chi + \hat{K}_\psi. \qquad (14)$$

These components can be interpreted in a physically meaningful way. If $\mathbf{V}_0$ is combined with $\hat{\mathbf{V}}_\psi$, the partitioning is the same as that obtained using versions 1 or 5 (i.e., those with $\chi_B = 0$). If $\mathbf{V}_0$ is combined instead with $\hat{\mathbf{V}}_\chi$, the result is the same as that of versions 3 or 7 (with $\psi_B = 0$).

If the domain $\Omega$ extends to the entire sphere, both $\chi_0$ and $\psi_0$ must be constant, since no other harmonic function is globally nonsingular (Carrier et al. 1966). Thus $\mathbf{V}_0$ and $K_0$ vanish and the partitioning of the wind

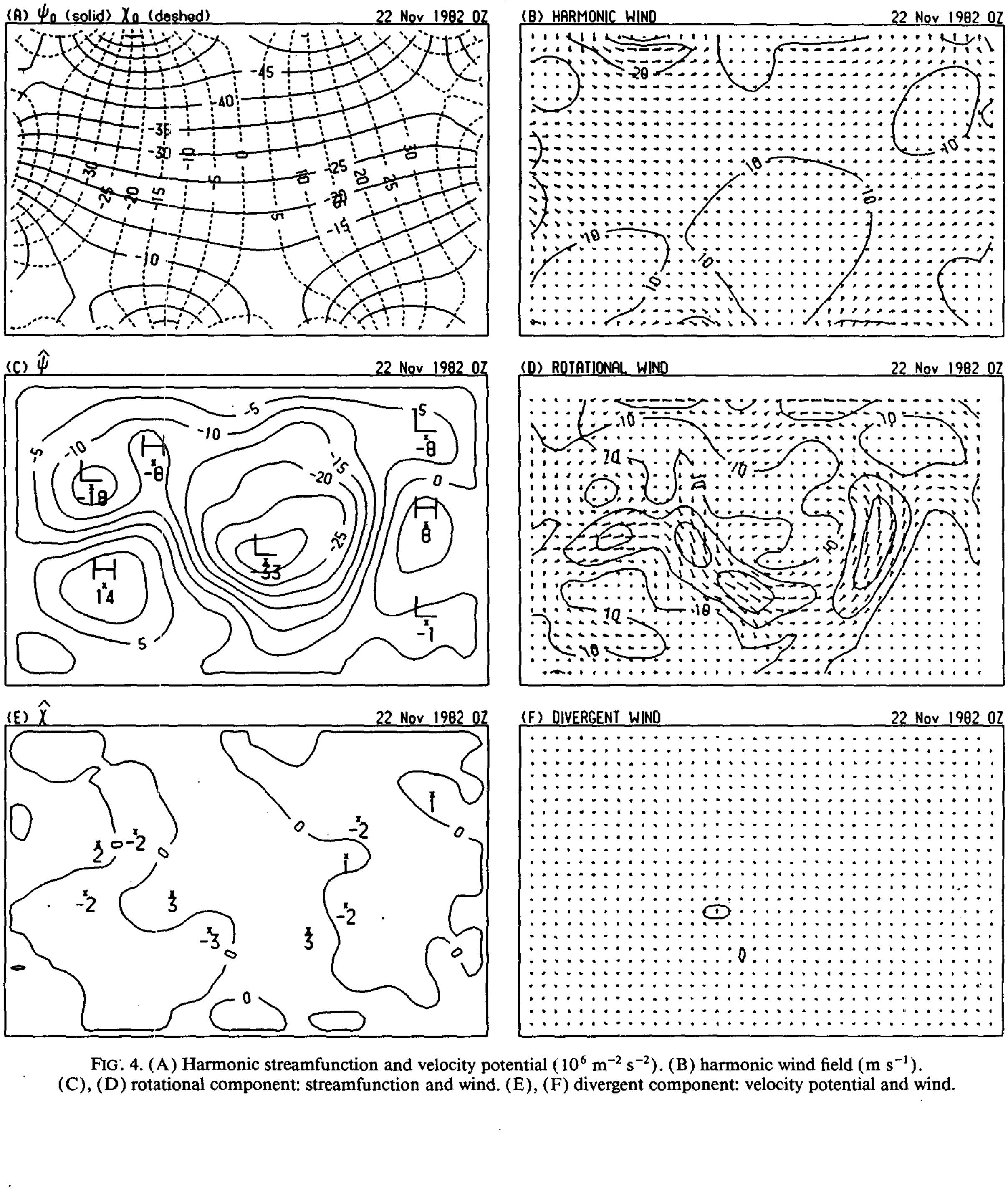


FIG. 4. (A) Harmonic streamfunction and velocity potential ($10^6$ m$^{-2}$ s$^{-2}$). (B) harmonic wind field (m s$^{-1}$). (C), (D) rotational component: streamfunction and wind. (E), (F) divergent component: velocity potential and wind.

and kinetic energy reduce to the usual two-component form.

An alternative partitioning of the wind into three components was described by Batchelor (1967, §2.4). He defined a streamfunction and velocity potential in terms of integrals involving the fundamental solution $1/r$ (in two dimensions it would be $\log r$) of the Poisson equation. This is appropriate for an unbounded domain, in which case $1/r$ is a Green's function for the problem (Morse and Feshbach 1953, Chap. 7). It is not so natural for a bounded domain, where the Green's function depends on the form of the boundary conditions. The rotational and divergent components determined in this way are not in general orthogonal and the kinetic energy cannot be partitioned into positive-definite components. The residual harmonic component is determined by specifying the normal boundary wind (Batchelor 1967, §2.7).

If we denote by $G(\mathbf{r}, \mathbf{r}_0)$ the Green's function for the Poisson equation (3a) which vanishes on $\partial\Omega$, the general solution for Dirichlet conditions can be written as the sum of an areal and a contour integral:

$$\chi(\mathbf{r}) = \iint_{\Omega} \delta(\mathbf{r}_0) G(\mathbf{r}, \mathbf{r}_0) d\mathbf{r}_0 + \oint_{\partial\Omega} \chi_B(s) \frac{\partial G}{\partial n} ds. \tag{15}$$

A similar expression holds for $\psi$. This is the natural analogue of Batchelor's solution for a limited domain. If $\chi_B = 0$, the solution given by the areal integral is identical to that obtained by solution version 1 above. Since $\chi_B$ and $\psi_B$ are not determined by the data, some such assumption must be introduced.

It is unclear whether the particular partitioning chosen by Batchelor has any characteristics which make it attractive for a limited area. The alternative proposed in this section has the advantage that the components are orthogonal, the splitting of kinetic energy is well defined and the minimal rotational and divergent components are isolated.

### *b. Numerical example*

The windfield already considered was partitioned into three components as described above. The resulting fields are plotted in Fig. 4. The harmonic stream function $\psi_0$ and velocity potential $\chi_0$ are shown in Fig. 4A. The isopleths can be seen to form two orthogonal families. The dominant structure of the harmonic wind (Fig. 4B) is a fairly uniform westerly flow of strength about 10 m s$^{-1}$. There are smaller features associated with strong cross-boundary flow in the original wind (Fig. 2B). This normal flow cannot project onto the rotational component because of the assumption $\hat{\psi}_B = 0$. The stream function $\hat{\psi}$ is shown in Fig. 4C and the rotational wind $\hat{\mathbf{V}}_\psi$ in Fig. 4D. The meandering jetstream projects mainly onto $\hat{\mathbf{V}}_\psi$, its overall strength lessened by the extraction of the harmonic component. The $\hat{\psi}$ field is identical to that obtained in version 3 above (Fig. 3E). The velocity potential $\hat{\chi}$, in Fig. 4E, is the same as that calculated in version 1 (Fig. 3B). The corresponding divergent wind $\hat{\mathbf{V}}_\chi$ is generally weak, with maxima of about 10 m s$^{-1}$ at the jet core.

TABLE 3. Components of the kinetic energy resulting from a three way splitting of the wind field, in units m$^2$ s$^{-2}$ (and as a percent of the total).

| | $V_n$ on $\partial\Omega$ | $V_s$ on $\partial\Omega$ |
|---|---|---|
| $K_0$ | 74.76 (39.5%) | 70.07 (36.5%) |
| $\hat{K}_\psi$ | 110.27 (58.2%) | 118.05 (61.4%) |
| $\hat{K}_\chi$ | 4.41 (2.3%) | 4.10 (2.1%) |
| $K_{\text{total}}$ | 189.44 (100.0%) | 192.22 (100.0%) |

The results in Fig. 4 were obtained using the normal boundary wind. The corresponding results using $V_s$ are not shown, as they do not differ in any essential way from those already discussed. However, the kinetic energy components for both cases are given in Table 3. The rotational component $\hat{K}_\psi$ dominates, but a sizeable part (about 40%) of the energy resides in the harmonic component $K_0$. The divergent wind contains only about 2% of the total. $\hat{K}_\psi$ and $\hat{K}_\chi$ are respectively the minimum values of rotational and divergent kinetic energy. The quantities $(\hat{K}_\psi + K_0)$ and $(\hat{K}_\chi + K_0)$ represent their maximum possible values. The harmonic component $K_0$ is that part of the energy which cannot be unambiguously designated as rotational or divergent without knowledge of the global winds.

## 4. Summary

The partitioning of a global windfield into rotational and divergent components is a useful diagnostic technique. For data on a limited domain these components are not uniquely defined. Several possible boundary conditions have been tried and the corresponding winds and energy components calculated. Constant Neumann conditions on either $\chi$ or $\psi$ (even-numbered versions in Table 1) result in a nonvanishing cross-term $K_{\chi\psi}$ in the kinetic energy, which is physically unsatisfactory. Constant Dirichlet conditions (odd-numbered versions) are free from this drawback.

In middle and high latitudes the stream function (deduced from global data) is highly correlated with the geopotential field. This similarity can be reflected in a two-way partitioning on a limited domain by minimizing the difference between the rotational and geostrophic winds (Davies-Jones 1988). In the tropics, such an approach is inappropriate.

A partitioning into three components allows us to minimize the rotational and divergent kinetic energy and to isolate that component of the flow which is both irrotational and nondivergent. The rotational, divergent and harmonic components are defined in an unambiguous way. Such a partitioning may be advantageous for diagnostic analysis of the wind on a limited domain.

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