Irish Math. Soc. Bulletin
Number 85, Summer 2020, 47–48
ISSN 0791-5578

Annalisa Crannell, Marc Frantz and Fumiko Futamura: Perspective and Projective Geometry, Princeton University Press, 2019. ISBN:9780-6911-9655-8, USD 49.95, x+280 pp.

REVIEWED BY PETER LYNCH

This book, essentially a course text, has lofty aspirations. It opens with the claim that it will "change the way that you look at the world, and we mean that literally". Seeing the world in new ways is mind-expanding and empowering, and this course may open the doors of perception just a little wider.

The challenge addressed in the book is to elucidate techniques used in graphical art by revealing the geometric principles underlying them. These techniques emerged from the Italian Renaissance and enabled artists to create strikingly realistic images. Among the most notable were Piero della Francesca and Leon Battista Alberti, who invented the method of perspective drawing. Artists were ahead of mathematicians, who only later codified the techniques in projective geometry. But the relationship became symbiotic, with each group learning from and teaching the other.

The book comprises twelve chapters and three appendices. For centuries, artists have painted scenes on a sheet of glass. In the opening lesson, students stick masking-tape on a large window, guided by an "artist", whose head is held in a fixed position. They discover that some lines that are parallel in the physical scene converge when marked on the window. There are many questions and exercises for the students to develop the capacity to visualize on a plane scenes in space, and many practical exercises where they must make drawings or take and use photographs, usually working together in groups.

Chapter 2 includes exercises on drawing large block letters in three dimensions. Chapter 3 answers the question "What is the image of a line?" The basics of Euclidean geometry are introduced in Chapter 4, and several theorems relevant to perspective are presented. In most cases, gaps are left in the proofs; this is deliberate and is intended to lead the students to make discoveries themselves. Students are asked to prove Ceva's Theorem and Menelaus's Theorem. Although generous hints are given, this will be daunting for many.

Chapter 5 introduces extended Euclidean space, with points and lines at infinity. This removes the need for special consideration of non-generic cases. In the context of perspective, the extended space includes the vanishing points of parallel lines. Chapter 6 discusses meshes and maps. Chapter 7 introduces Desargues's Theorem. Once again, the proof is gappy. This reviewer suspects that most readers will not have the tenacity (or inclination) to work through all the proofs and fill in all the gaps.

Collineations are considered in Chapter 8. The following two chapters are about drawing boxes and cubes. In Chapter 11, the cross-ratio, of key importance in both perspective drawing and projective geometry, is defined and applied. Eve's Theorem is proved (in outline) and Casey's Angle, a projective invariant, is introduced and illustrated by application to a perspective drawing. The invariance of this angle was first proved by the Irish geometer John Casey. I found the discussion in this chapter

Received on 7-4-2020.

less lucid than elsewhere in the book, and somewhat confusing. Students will probably have the same reaction.

Chapter 12, on coordinate geometry, is more algebraic in character, enabling us to do perspective "by the numbers". The final chapter is on arcana like the topology of the perspective plane and the shape of space in the large. It is more abstract and less likely to assist artists directly, but perhaps it will give them a "broader perspective on perspective".

There are three appendices. The first gives complete beginners an introduction to the GeoGebra graphics program. The second, for reference, collects all the main definitions and results in one place. The third, on writing mathematical prose, has practical advice that should benefit all students. A Bibliography with more than fifty references and a two-page Index conclude the book.

⋆ ⋆ ⋆

It is difficult to know precisely who will benefit most from this book. There will be little new to mathematicians interested in projective geometry. And most artists will be strongly deterred by symbolic formulations of the type $\exists P \in \mathbb{R}^3 : \mathcal{P} = \{p \subset \mathbb{R}^3 : P \in p\}$ occurring in the definition of a pencil of lines. Students will need to work hard to benefit from the course. The absence of solutions means that they will struggle unless they have the guidance of an instructor, so the book is not really suitable for self-study except by especially talented readers.

The authors have undertaken a formidable task: to teach mathematics (geometry) to artists and (graphic) art to geometers. They have been only partially successful. They are not the first to struggle with such a task. We recall Euler's book *Tentamen novæ theoræ musicæ*, completed when he was 23 years old. This work was described as "too mathematical for musicians and too musical for mathematicians".

Peter Lynch is emeritus professor at UCD. His interests include all areas of mathematics and its history. He writes an occasional mathematics column in The Irish Times and has published a book of articles entitled *Thats Maths.* His blog is at http://thatsmaths.com.

School of Mathematics & Statistics, University College Dublin
E-mail address: Peter.Lynch@ucd.ie